Properties of Logarithms

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Product Rule: $\log_b(MN) = \log_b M + \log_b N$

Quotient Rule: $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

Power Rule: $\log_b M^p = p\log_b M$

Ex 1.

Use properties of logarithms to expand each expression.

$\log_7(8 \cdot 6) =$

$\log 100x =$

$\ln\left(\frac{e^3}{13}\right) =$

$\log_6 8^9 =$

$\log_2 \sqrt[3]{x} =$

$\log(x+4)^2 =$

$\log_b x^4\sqrt{y} =$

$\log_5 \frac{\sqrt[3]{x}}{25y^3} =$

Ex 2.
Write each expression as a single logarithm.

$\log(4x - 3) - \log x =$

$\log 25 + \log 4 =$

$\frac{1}{4}\log_b x - 2\log_b 5 - 10\log_b y =$

Change-of-Base Theorem

$$\log_b M = \frac{\log_a M}{\log_a b}$$

Ex 3.
Use the Change-of-Base Theorem to rewrite $\log_5 17$ in terms of natural logarithms. Then use a calculator to evaluate to four decimal places.

Ex 4.
Use the Change-of-Base Theorem to rewrite $\log_2 0.3$ in terms of common logarithms. Then use a calculator to evaluate to four decimal places.

Practice

1. Use properties of logarithms to expand each expression. Simplify where possible.

a) $\log_9(9x)$

b) $\log\left(\frac{x}{1000}\right)$

c) $\log_b x^7$

d) $\log_8\left(\frac{64}{\sqrt{x+1}}\right)$

e) $\log_b\left(\frac{\sqrt[3]{x}y^4}{z^5}\right)$

2. Write each expression as a single logarithm. Simplify where possible.

a) $\log 250 + \log 4$

b) $\log_3 405 - \log_3 5$

c) $\log x + 7\log y$

d) $8\ln(x+9)-4\ln x$

3. Use the Change-of-Base Theorem to rewrite $\log_4 23$ in terms of natural logarithms. Then use a calculator to evaluate to four decimals places.

Q: What do you get when you expand $(x-a)(x-b)(x-c)\ldots(x-y)(x-z)$?