Logarithmic Functions

Logarithmic Function

One-to-one functions have inverses. Let's define the inverse of the exponential function.

To find inverse of $y = b^x$, we first switch x and y: $x = b^y$ Then we solve for y.

We don't have tools to solve for y, so we just define what's called the _____:

$\boldsymbol{y = \log_b x}$

(Note: Here, b and x are positive, and $b \neq 1$.)

Logarithmic Form: $\boldsymbol{\log_b x = y}$ Exponential Form: $\boldsymbol{b^y = x}$

Ex 1.

Write each equation in the other form.

Exponential Form	Logarithmic Form
	$3 = \log_7 x$
	$\log_4 26 = y$
$2^5 = x$	
$b^3 = 27$	

Ex 2.

Evaluate.

$\log_2 8 =$ $\log_3 3 =$

$\log_e \frac{1}{e^3} =$ $\log_{36} 6 =$

Note: $\boldsymbol{\log_b b =}$ ____ and $\boldsymbol{\log_b 1 =}$ ____

Note: Since b^x and $\log_b x$ are inverse functions by definition, $\boldsymbol{b^{\log_b x} =}$ _____ and $\boldsymbol{\log_b b^x =}$ _____

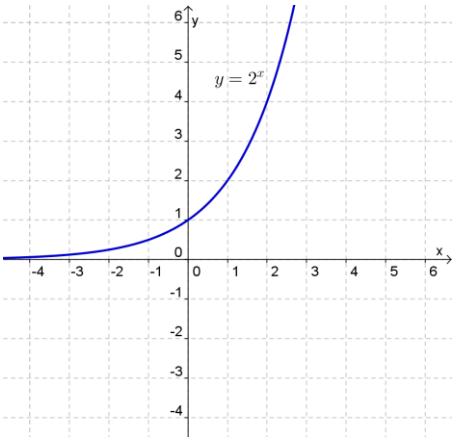
Ex 3.

Evaluate.

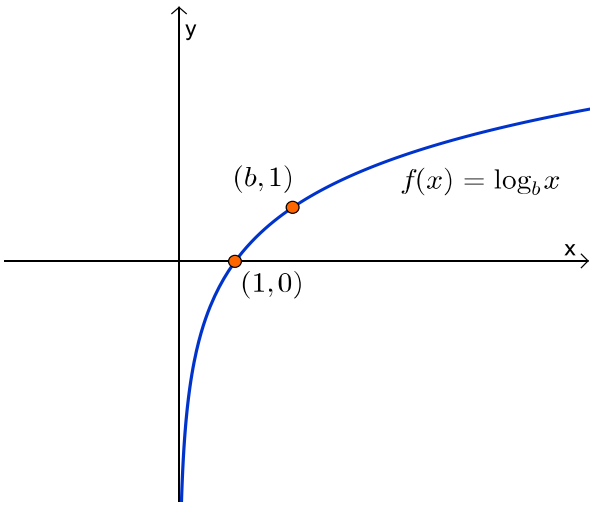
$\log_7 7^8 =$ $3^{\log_3 7} =$

Graph of Logarithmic Function

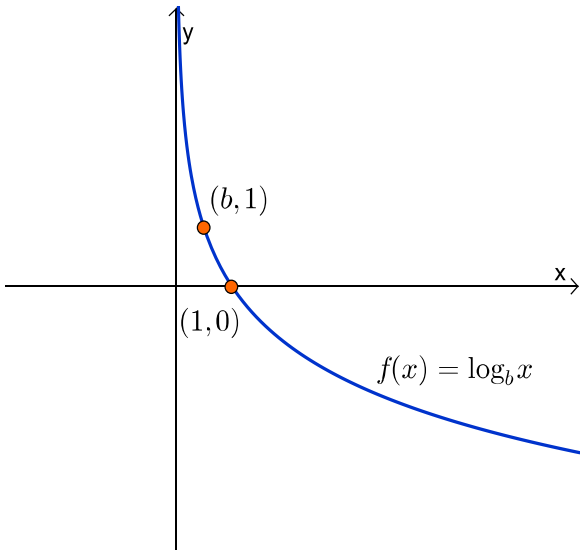
Let's graph $y = \log_2 x$, knowing that it's the inverse of 2^x:



In general, the **logarithmic function with base b** is $\boldsymbol{f(x) = \log_b x}$ (where $b > 0$, $b \neq 1$, $x > 0$).



$\boldsymbol{b > 1}$
ex: $f(x) = \log_3 x$



$\boldsymbol{0 < b < 1}$
ex: $f(x) = \log_{1/3} x$

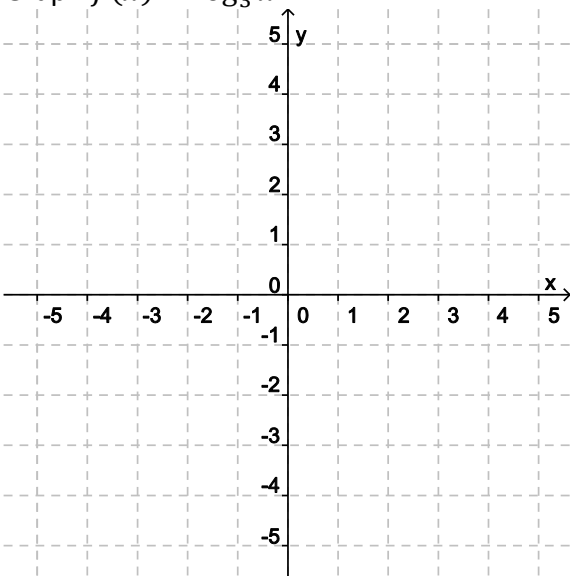
What is the vertical asymptote? _____

What is the domain? _____

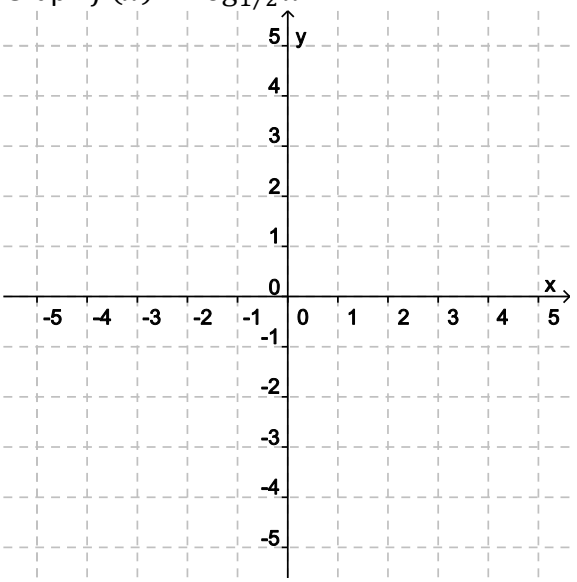
What is the range? _____

Ex 4.

Graph $f(x) = \log_3 x$



Graph $f(x) = \log_{1/2} x$



Ex 5.

What is the domain of $f(x) = \log_4(x - 5)$?

Notation:

$\mathbf{\log x = \log_{10} x}$ (_____ logarithm)

$\mathbf{\ln x = \log_e x}$ (_____ logarithm)

Ex 6.

$\ln e^3 =$

$\log 1000 =$

Practice

1. Write in exponential form.

a) $2 = \log_9 x$

b) $\log_5 125 = y$

2. Write in logarithmic form.

a) $2^{-4} = \frac{1}{16}$

b) $8^y = 300$

3. Evaluate.

a) $\log_3 27$

b) $\log_5 \frac{1}{\sqrt{5}}$

c) $\log 100000$

d) $\log_7 1$

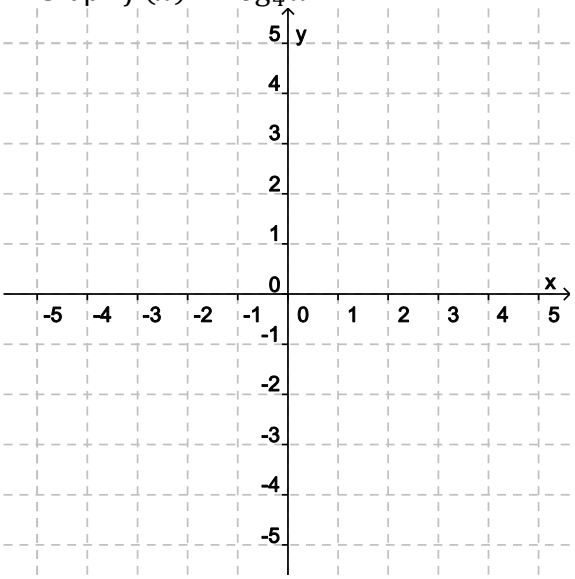
e) $\log_x x^4$

f) $6^{\log_6 18}$

g) $e^{\ln 7x^2}$

h) $\log 10^{99}$

4. Graph $f(x) = \log_4 x$



5. What is the domain of $f(x) = \ln(7 - x)$?

Q: A man goes into a bar and asks for a glass of water. The barman pulls out a gun, and points it at the customer. "Thank you" replies the customer and walks out. What happened?