

Composite and Inverse Functions

When you plug a function into another function, it's called _____.

ex: If $f(x) = x^2$ and $g(x) = 3x - 2$, then $f(g(x)) =$ _____

Notation: $(f \circ g)(x) =$ _____

The domain of $f \circ g$ is the set of all ____ in the domain of ____ such that ____ is in the domain of ____.



Ex 1.

Let $f(x) = 5x + 6$ and $g(x) = x^2 - 1$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$.

Some functions “undo” each other. Like $f(x) = x + 5$ and $g(x) = x - 5$.
Functions like this are called _____.

What happens when we compose inverses? Let's try with our above functions:

$$f(g(x)) =$$

$$g(f(x)) =$$

Here's the formal definition: f and g are **inverse functions** if both

1. $f(g(x)) =$ _____ (for every x in the domain of g)
2. $g(f(x)) =$ _____ (for every x in the domain of f)

Notation: The inverse of f is written f^{-1} .

Ex 2.

Show that $f(x) = 4x - 7$ and $g(x) = \frac{x+7}{4}$ are inverses of each other.

How to find the inverse of $f(x)$:

1. Replace $f(x)$ with ____.
2. _____ x and y .
3. Solve for ____.
4. Replace y with _____.

Ex 3.

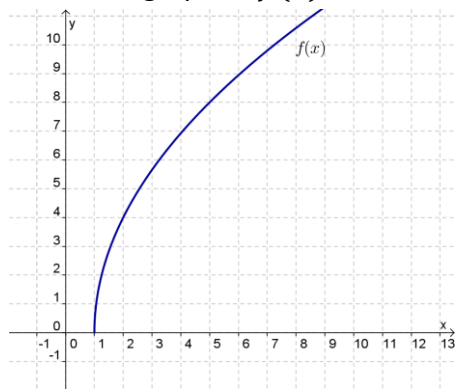
Find the inverse of $f(x) = 4x^3 - 1$

Graphs and Inverses

Given any point (a, b) on the graph of $f(x)$, we can get a point on the graph of $f^{-1}(x)$ by switching the coordinates: (b, a) .

Ex 4.

Given the graph of $f(x)$, draw the graph of $f^{-1}(x)$.

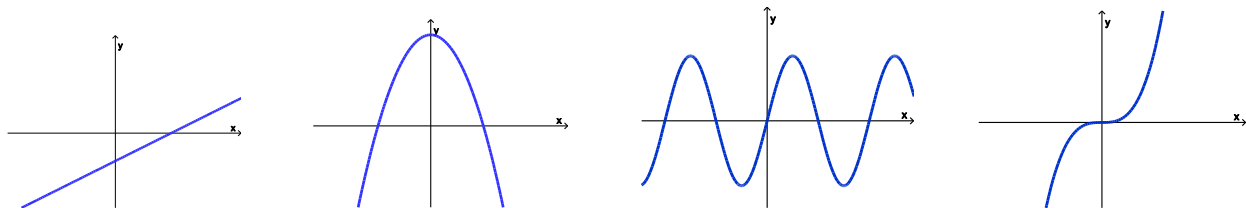


Notice that the entire graph of $f^{-1}(x)$ will be the mirror image of $f(x)$ across the line $y = x$.

The _____ is a visual way to determine if a function has an inverse.

Ex 5.

Do the following graphs have inverses?



Note: Functions that pass the horizontal line test are called _____ functions.

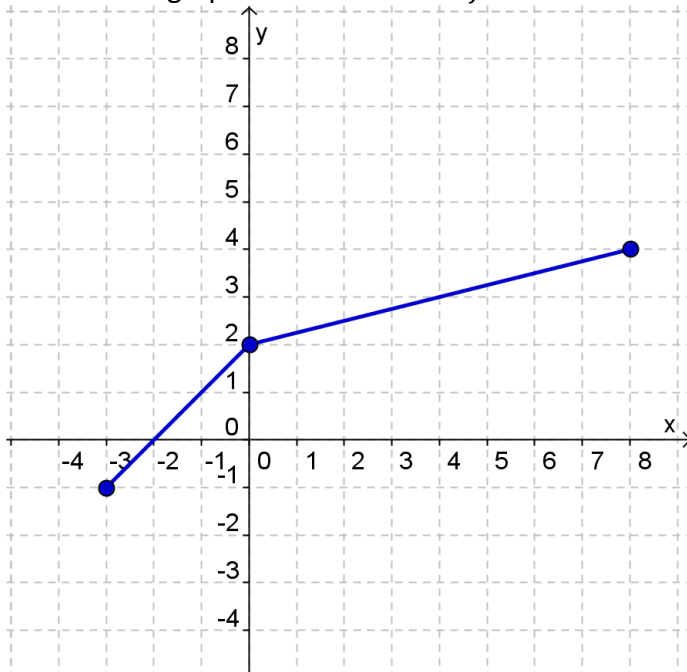
Practice

1. Let $f(x) = \sqrt{x}$ and $g(x) = x + 2$. Find $(f \circ g)(x)$, $(g \circ f)(x)$, and $(f \circ g)(2)$

2. Are $f(x) = \frac{3}{x-4}$ and $g(x) = \frac{3}{x} + 4$ inverses of each other? Show by finding $f(g(x))$ and $g(f(x))$.

3. Find the inverse of $f(x) = 3x - 1$.

4. Draw the graph of the inverse of f .



Q: A man in a café has coffee and a fly flies into it. He calls the waiter over, who takes the coffee to the kitchen, and a few moments later returns with another cup of coffee. The man says, "This is the same cup of coffee--you just took the fly out!" How could he know?