

Exponential Functions

Recall:

$$b^{m/n} = (\sqrt[n]{b})^m = \sqrt[n]{b^m}$$

$$4^{1.5} =$$

It turns out that exponents can be any real # (rational or **irrational**).

$$\text{ex: } 2^{\sqrt{3}} = 3.321997 \dots$$

We can get this by taking closer and closer approximations to $\sqrt{3}$ (which is about 1.732...):

$$2^{1.7} = 2^{17/10} = (\sqrt[10]{2})^{17} = 3.249010 \dots$$

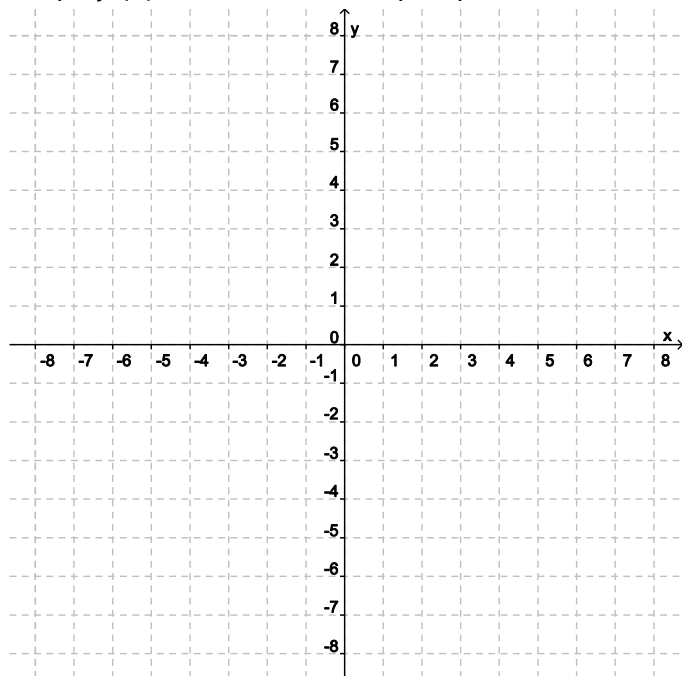
$$2^{1.73} = 2^{173/100} = (\sqrt[100]{2})^{173} = 3.317278 \dots$$

$$2^{1.732} = 2^{1732/1000} = (\sqrt[1000]{2})^{1732} = 3.321880 \dots$$

So, since we can plug any real number into 2^x , we can define a function $f(x) = 2^x$ that has a domain of all real numbers.

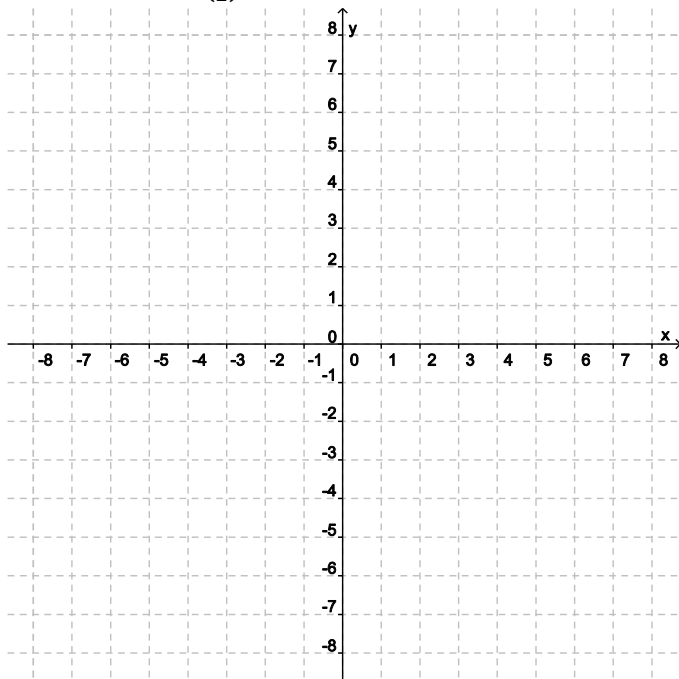
Ex 1.

Graph $f(x) = 2^x$. Be sure to plot points at $x = -2, -1, 0, 1,$ and 2 .

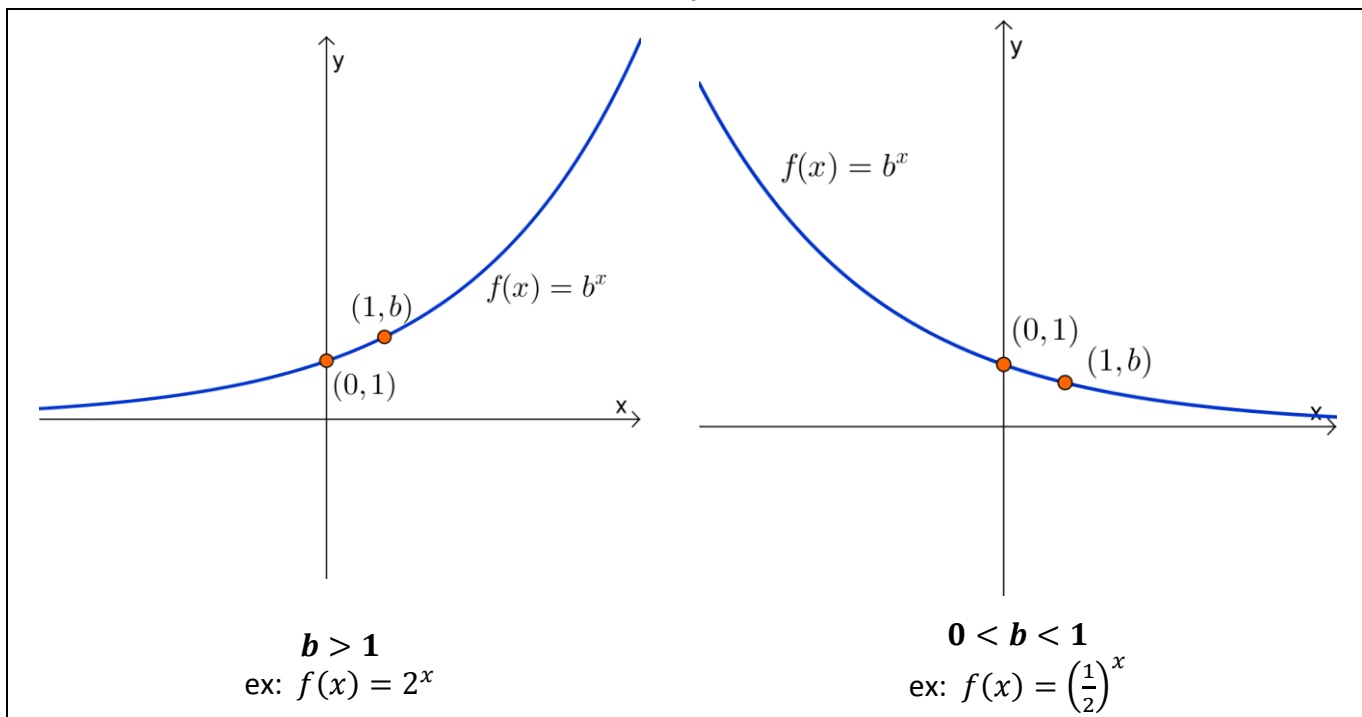


Ex 2.

Graph $f(x) = \left(\frac{1}{2}\right)^x$. Be sure to plot points at $x = -2, -1, 0, 1,$ and 2 .



In general, the **exponential function with base b** is $f(x) = b^x$ (where $b > 0$ and $b \neq 1$).



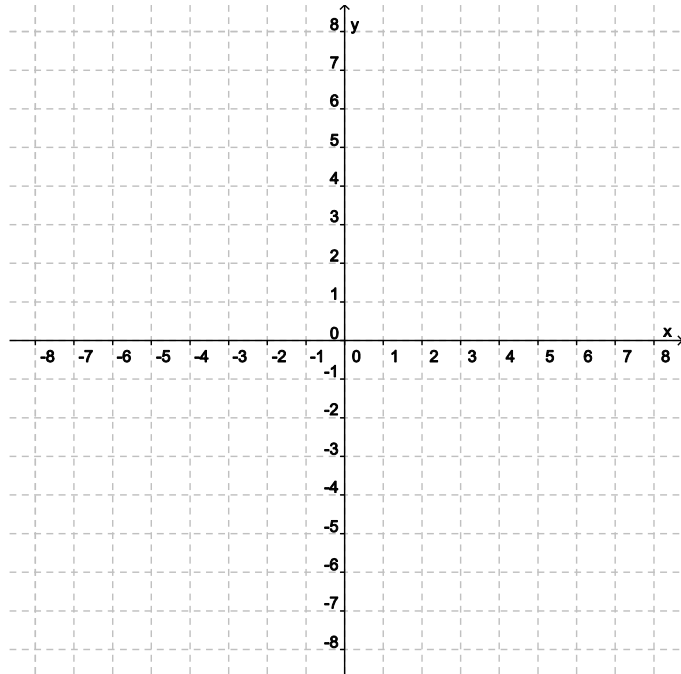
What is the domain? _____

What is the range? _____

What is the horizontal asymptote? _____

Ex 3.

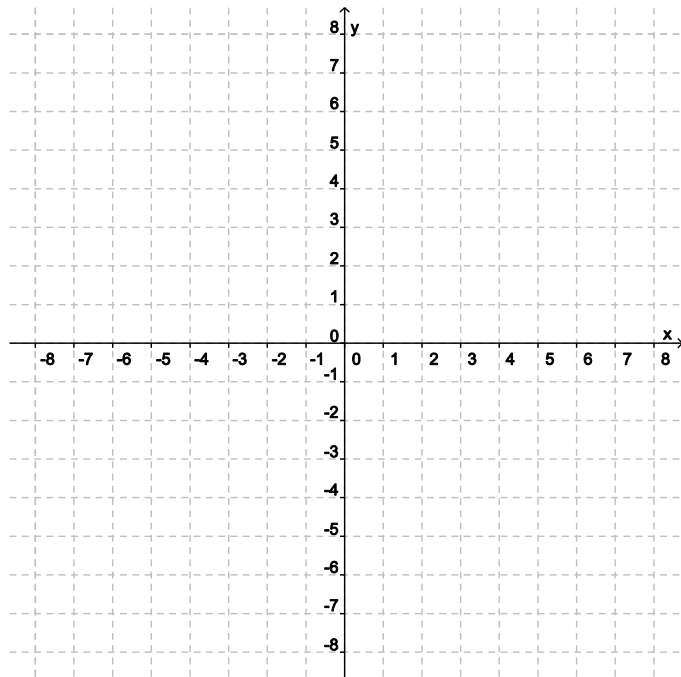
Graph $f(x) = 2^x$ and $g(x) = 2^x - 5$ in the same rectangular coordinate system.



How are the graphs related? _____

Ex 4.

Graph $f(x) = 3^x$ and $g(x) = 3^{x+1}$ in the same rectangular coordinate system.



How are the graphs related? _____

Natural Base e

e (called the natural base) is defined to be the number that $\left(1 + \frac{1}{n}\right)^n$ approaches as n gets larger and larger.

| n | $\left(1 + \frac{1}{n}\right)^n$ |
|-----------|----------------------------------|
| 1 | 2 |
| 2 | 2.25 |
| 5 | 2.48832 |
| 10 | 2.59374 |
| 100 | 2.70481 |
| 1,000 | 2.71692 |
| 1,000,000 | 2.71828 |

Continuing, we get that e is an irrational # 2.718281828459045...

Note: The graph of e^x is between the graphs of 2^x and 3^x . (Since $2 < e < 3$.)

Calculators

Here's how to use the TI-30XIIS to evaluate exponentials:

ex: $3^{1.5}$

1. Enter 3
2. Push “^” button
3. Enter 1.5
4. Push =

You should get approximately 5.196152.

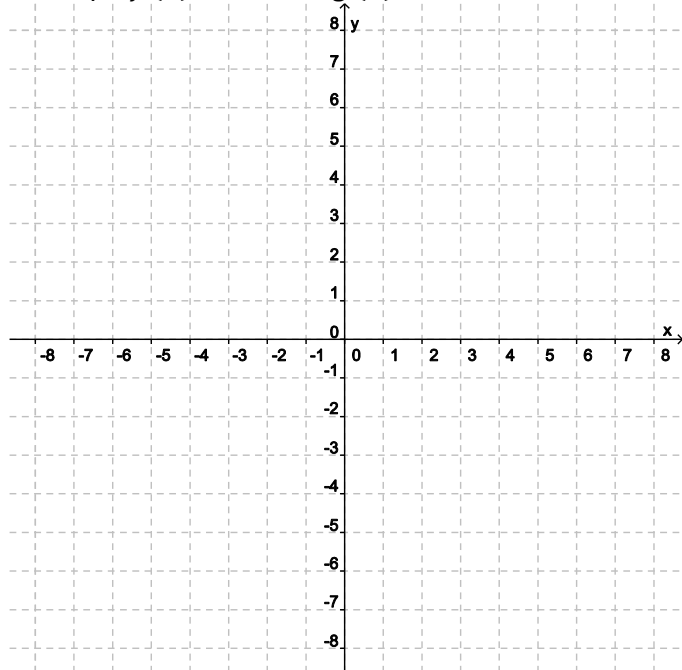
ex: $e^{1.5}$

1. Push 2nd button
2. Push LN/ e^x button
3. Enter 1.5
4. Push =

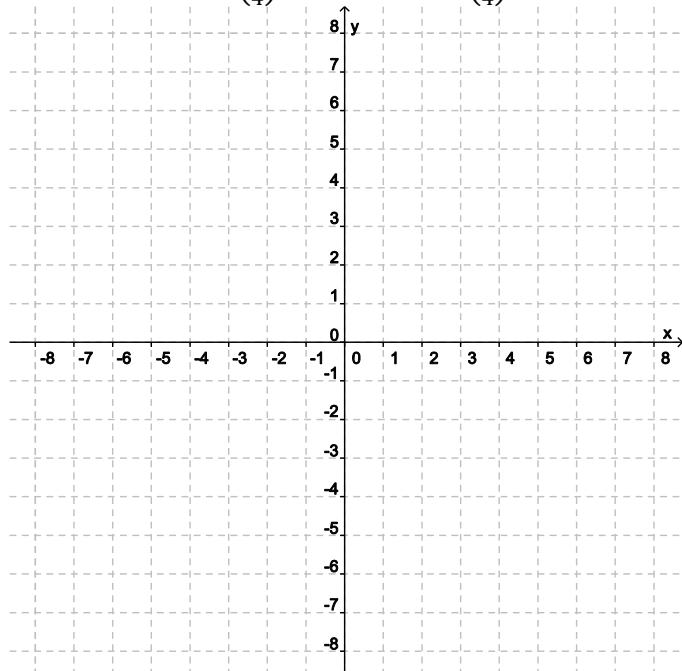
You should get approximately 4.481689.

Practice

1. Graph $f(x) = 4^x$ and $g(x) = 4^x + 2$. How is the graph of g related to the graph of f ?



2. Graph $f(x) = \left(\frac{1}{4}\right)^x$ and $g(x) = \left(\frac{1}{4}\right)^{x-3}$. How is the graph of g related to the graph of f ?



3. Approximate each number using a calculator. Round your answer to three decimal places.

a) $3^{2.4}$

b) $e^{3.4}$

c) $e^{-0.75}$

Q: April says May is a liar. May says June is a liar. June says April and May are both liars. If only one person is telling the truth, who is it?