

Equations Quadratic in Form

An equation is **quadratic in form** if you can make a substitution (for example, $u = x^{\frac{1}{3}}$) that turns the equation into a quadratic equation.

ex: $x^4 - 5x^2 + 4 = 0$ is quadratic in form, since you could make the substitution $u = x^2$ to get $u^2 - 5u + 4 = 0$, which is a quadratic equation.

Ex 1.

Solve: $x^4 - 5x^2 + 6 = 0$

Ex 2.

Solve: $2x^{-2} + x^{-1} - 1 = 0$

Ex 3.

Solve: $x - 2\sqrt{x} - 8 = 0$

Practice

1. Solve: $x^4 + 4x^2 = 5$ (Hint: move 5 to LHS first. Also, include any complex solutions.)

2. Solve: $x - 6\sqrt{x} + 8 = 0$

3. Solve: $x^{\frac{2}{3}} + 2x^{\frac{1}{3}} - 3 = 0$

4. Solve: $(x^2 - 2)^2 - (x^2 - 2) = 6$ (Hint: let $u = x^2 - 2$)

Q: A man wanted to enter an exclusive club but did not know the password that was required. He waited by the door and listened. A club member knocked on the door and the doorman said, "twelve." The member replied, "six," and was let in. A second member came to the door and the doorman said, "six." The member replied, "three," and was let in. The man thought he had heard enough and walked up to the door. The doorman said, "ten," and the man replied, "five." But he was not let in. What should he have said?