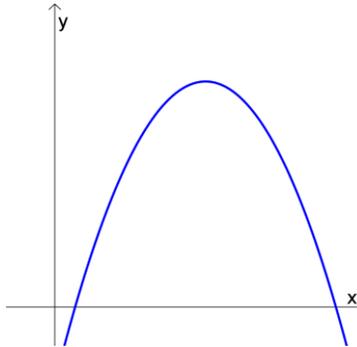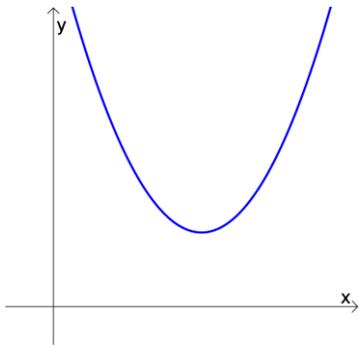# Quadratic Functions and Their Graphs

Recall: A **quadratic function** can be written in the form $f(x) = ax^2 + bx + c$

The graph of a quadratic function is a _____. Two important features that distinguish

parabolas are the _____ and the _____.

For bigger values of a, the y-values get bigger faster, so the graph is steeper.
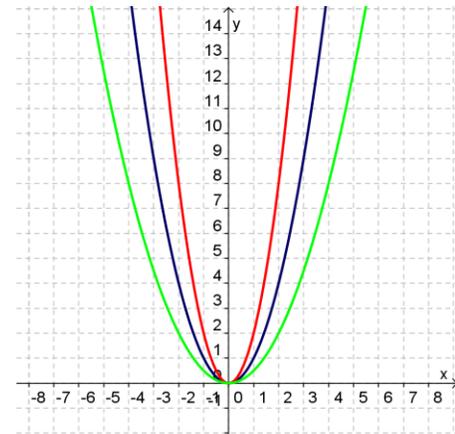
For smaller values of a, the y-values get bigger more slowly, so the graph is flatter.

To the right are the graphs of ax^2 when $a = \frac{1}{2}, 1$, and 2.

Any quadratic function can be rewritten in the form
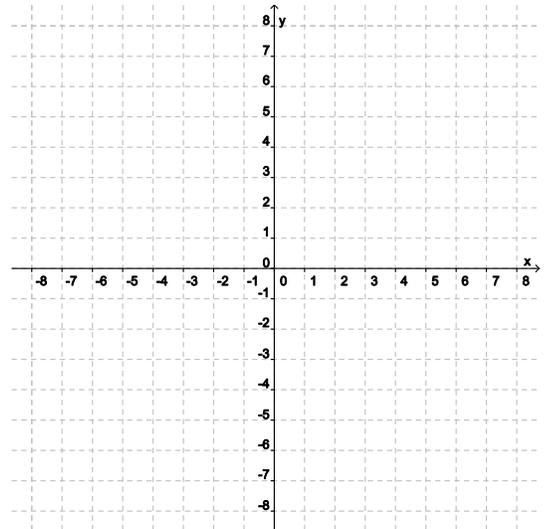
$$f(x) = a(x - h)^2 + k$$

The vertex is _____ and the axis of symmetry is _____.
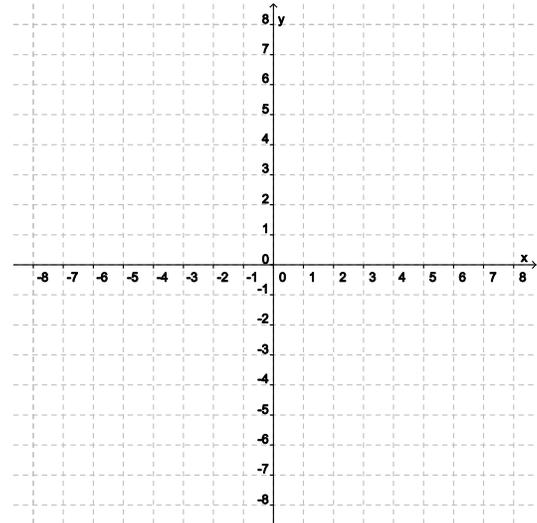
Ex 1.

Graph $f(x) = -(x - 1)^2 + 4$.

1. Does $f(x)$ open up or down? _____

2. What is the vertex of $f(x)$? _____

3. What are the x-intercepts?

4. What is the y-intercept?

Ex 2.

Graph $f(x) = (x+2)^2 + 1$.

1. Does $f(x)$ open up or down? _____

2. What is the vertex of $f(x)$? _____

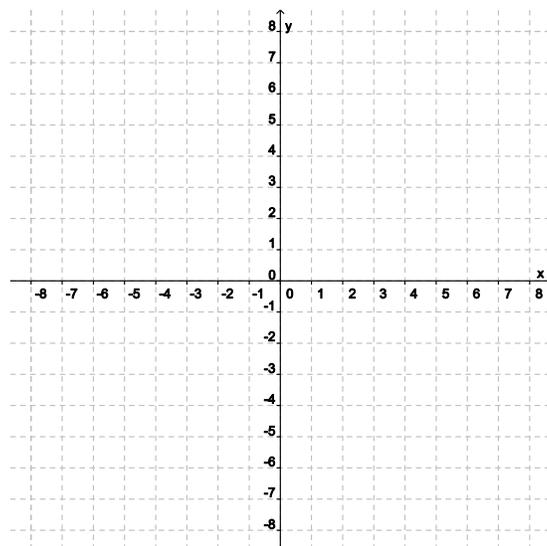3. What are the x-intercepts?

4. What is the y-intercept?

Vertex of $f(x) = ax^2 + bx + c$

We can complete the square to turn $ax^2 + bx + c$ into the form $a(x-h)^2 + k$ to get vertex:

$$\begin{aligned}
f(x) &= ax^2 + bx + c \\
&= a\left(x^2 + \frac{b}{a}x\right) + c \\
&= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - a\left(\frac{b^2}{4a^2}\right) \\
&= a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} \\
&= a\left(x - \underbrace{\left(-\frac{b}{2a}\right)}_{h}\right)^2 + \underbrace{c - \frac{b^2}{4a}}_{k}
\end{aligned}$$

So, the vertex is $\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$, but we really only need to remember that the x-value is $-\frac{b}{2a}$...

Vertex of $f(x) = ax^2 + bx + c$ is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

Ex 3.

Graph $f(x) = -x^2 + 4x + 1$.

1. Does $f(x)$ open up or down? _____

2. What is the vertex of $f(x)$?

3. What are the x-intercepts?

4. What is the y-intercept?

Minimum/Maximum Values

If $a > 0$, then a _____ occurs at $x = -\frac{b}{2a}$, and the _____ is $f\left(-\frac{b}{2a}\right)$.

If $a < 0$, then a _____ occurs at $x = -\frac{b}{2a}$, and the _____ is $f\left(-\frac{b}{2a}\right)$.

Ex 4.

Does $f(x) = -3x^2 + 6x - 13$ have a minimum or maximum?

At what x-value does the minimum or maximum occur?

What is the minimum or maximum value?

What are the domain and range of $f(x)$?

Ex 5.

If you can buy 120 feet of fencing to enclose a rectangular region, what are the dimensions of the rectangle that maximize the enclosed area? Also, what is the maximum area?

Practice

1. Find the coordinates of the vertex for $f(x) = -3(x-2)^2 + 12$. Also, what is the axis of symmetry?

2. Find the coordinates of the vertex for $f(x) = 3x^2 - 12x + 1$. Also, what is the axis of symmetry?

3. Graph $f(x) = x^2 - 4x + 6$. Find and plot the vertex and intercepts. Identify the range. Where does the maximum or minimum occur, and what is the minimum or maximum value?
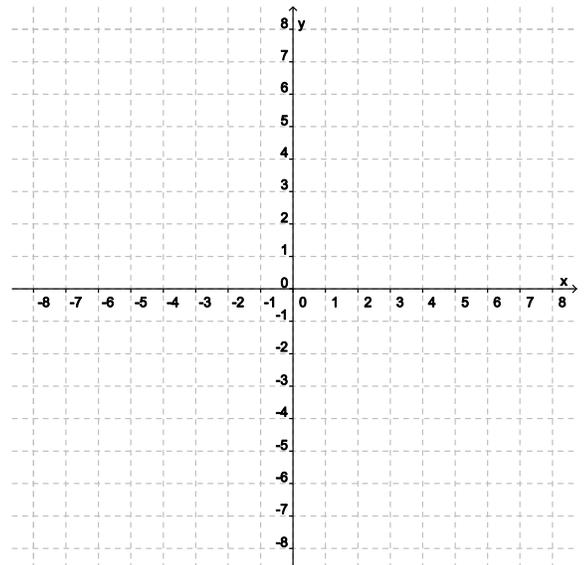
4. You have 200 feet of fencing to enclose a rectangular plot that borders on a river. If you do not fence the side along the river, find the length and width of the plot that will maximize the area. What is the largest area that can be enclosed?

X

X

$200 - 2x$
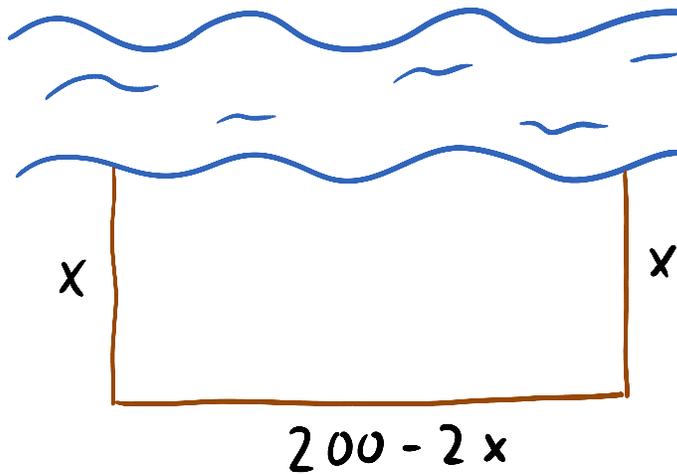
5. Find two numbers add to 20, and have the largest product possible. Also, what is the maximum product?

Q: If you have 10 black socks and 10 green socks in a drawer (all mixed up), how many socks do you need to take out to be sure that you have a pair of the same color?