

The Quadratic Formula

Let's use the completing the square technique to derive a general formula to solve:

$$ax^2 + bx + c = 0.$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \leftarrow \text{The quadratic formula!}$$

Ex 1.

Solve using the quadratic formula: $2x^2 + 9x - 5 = 0$

Ex 2.

Solve using the quadratic formula: $2x^2 = 6x - 1$

Ex 3.

Solve using the quadratic formula: $3x^2 + 5 = -6x$

Discriminant

The **discriminant** of $ax^2 + bx + c$ is _____.

Suppose a , b , and c are rational numbers.

Discriminant	Kinds of solutions
$b^2 - 4ac > 0$	Two real solutions. If perfect square, then rational solutions. If not perfect square, then irrational solutions.
$b^2 - 4ac = 0$	One rational solution (repeated).
$b^2 - 4ac < 0$	Two imaginary solutions.

Ex 4.

Compute the discriminant. Then determine the number and type of solutions.

$$x^2 + 6x + 9 = 0$$

$$2x^2 - 7x - 4 = 0$$

$$3x^2 - 2x + 4 = 0$$

Practice

1. Solve using the quadratic formula.

a) $4x^2 + 3x = 2$

b) $x^2 + 6x + 13 = 0$

2. Compute the discriminant. Then determine the number and type of solutions.

a) $2x^2 - 4x + 3 = 0$

b) $x^2 + 7x + 4 = 0$

c) $4x^2 = 20x - 25$

Q: A sheik announced that a race would decide which of his two sons would inherit all his wealth. The sons were to ride their camels to a certain distant city. The son whose camel reached the city last would be given all of the sheik's wealth. The two sons set out on the journey. After several days of aimless wandering, they met and agreed to seek the advice of a wiseman. After listening to the wiseman's advice, the two sons rode the camels as quickly as possible to the designated city. They did not agree to split the wealth, and their father's decree was to be followed. What was it the wiseman told the two sons?