Rational Exponents

Let's define what it means to have a rational # as an exponent: $\boldsymbol{a^{\frac{1}{n}}} =$ ________

Ex 1.

$81^{\frac{1}{2}} =$

$(-64)^{1/3} =$

Note: We can write $a^{2/3}$ in a couple of different ways:

$a^{\frac{2}{3}} = \left(a^{\frac{1}{3}}\right)^2 = (\sqrt[3]{\boldsymbol{a}})^{\boldsymbol{2}}$ **or** $a^{\frac{2}{3}} = (a^2)^{\frac{1}{3}} = \sqrt[3]{\boldsymbol{a^2}}$

Radical Notation	**Rational Exponent**
$\sqrt[7]{x}$	
$\sqrt{x}$	
$\sqrt[3]{x^5}$	
$\sqrt{x^3}$	
	$x^{\frac{1}{8}}$
	$x^{\frac{1}{2}}$
	$x^{\frac{5}{6}}$
	$x^{\frac{7}{2}}$

Ex 2.

Use radical notation to rewrite each expression and simplify.

$8^{\frac{4}{3}} =$

$-81^{\frac{3}{4}} =$

Ex 3.

Rewrite with rational exponents:

$\sqrt[3]{5^4} =$

$\left(\sqrt[4]{11x^2y}\right)^9 =$

Ex 4.

Rewrite with a positive exponent. Simplify if possible.

$100^{-\frac{1}{2}} =$

$32^{-\frac{3}{5}} =$

$(3xy)^{-\frac{5}{9}} =$

Note: The same rules that you learned before apply when working with rational exponents.

$b^m \cdot b^n = b^{m+n}$ $\quad \frac{b^m}{b^n} = b^{m-n}$ $\quad (b^m)^n = b^{mn}$ $\quad (ab)^n = a^n b^n$ $\quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $\quad b^{-n} = \frac{1}{b^n}$ $\quad b^0 = 1$

When is an expression with rational exponents **simplified**?

Basically, when you're done using the above rules. That means…

1. No parentheses around products or quotients. ex: $(3x)^2$ or $\left(\frac{x}{3}\right)^2$
2. No powers raised to powers. ex: $(x^3)^5$
3. Each base occurs once. ex: x^2x^3
4. No negative or zero exponents. ex: x^{-6} and x^0

(Exception to "1": if final answer is in radical notation, try to bring factors into a common radical.

ex: $a^{\frac{1}{2}}b^{\frac{1}{2}} = (ab)^{\frac{1}{2}} = \boxed{\sqrt{ab}}$ not $a^{\frac{1}{2}}b^{\frac{1}{2}} = \sqrt{a}\sqrt{b}$)

Ex 5.

Simplify (assume all variables represent positive numbers):

$7^{\frac{1}{2}} \cdot 7^{\frac{1}{3}} =$

$\frac{50x^{\frac{1}{3}}}{10x^{\frac{4}{3}}} =$

$\left(x^{-\frac{3}{5}}y^{\frac{1}{4}}\right)^{\frac{1}{3}} =$

$\frac{\sqrt[5]{y^2}}{\sqrt[10]{y^3}} =$

$\sqrt{x} \cdot \sqrt[3]{x} =$

$\sqrt[3]{\sqrt{x}} =$

$\sqrt[6]{ab^2} \cdot \sqrt[3]{a^2b} =$

Practice

1. Use radical notation to rewrite each expression. Simplify if possible.

$(-32)^{\frac{1}{5}} =$

$(xy)^{\frac{1}{4}} =$

$25^{\frac{3}{2}} =$

2. Rewrite each expression with rational exponents.

$\sqrt{17} =$ $\sqrt[7]{x^4} =$ $\left(\sqrt{13x^2y}\right)^5 =$

3. Rewrite each expression with a positive rational exponent. Simplify if possible.

$125^{-\frac{1}{3}} =$ $32^{-\frac{4}{5}} =$

$7xz^{-\frac{1}{4}} =$ $(3xy)^{-\frac{2}{3}} =$

4. Simplify (assume all variables represent positive numbers).

$2^{\frac{2}{5}} \cdot 2^{\frac{3}{5}} =$

$\left(32^{\frac{2}{3}}\right)^{\frac{3}{5}} =$

$\frac{x^{\frac{1}{4}}}{x^{\frac{3}{5}}} =$

$\left(y^{-\frac{3}{4}}\right)^{\frac{1}{6}} =$

$\left(8x^{\frac{1}{4}}y^{-\frac{2}{5}}\right)^{\frac{1}{3}} =$

7.2 – Notes

5. Simplify (assume all variables represent positive numbers). Write answers in radical notation.

$\sqrt[5]{x^{15}y^{20}} =$

$\left(\sqrt[8]{2a}\right)^6 =$

$\sqrt[5]{\sqrt{x}} =$

$\dfrac{\sqrt[4]{a^3b^3}}{\sqrt{ab}} =$

Q: What is harder to catch the faster you run?