The Binomial Theorem

Binomial Expansion Pattern

$(a + b)^0 = 1$

$(a + b)^1 = a + b$

$(a + b)^2 = a^2 + 2ab + b^2$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

Looking at the coefficients of the terms, a pattern emerges…

Pascal's Triangle

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                1
            1       1
         1      2       1
       1     3      3      1
     1     4      6      4      1
   1     5    10     10     5      1
```

Binomial Coefficients

In general, the coefficients of the terms of the binomial expansion of $(a + b)^n$ are given by:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Ex 1.

Evaluate the following:

$$\binom{6}{2}$$

$$\binom{9}{6}$$

$$\binom{7}{0}$$

$$\binom{4}{4}$$

Binomial Theorem

The Binomial Theorem tells us how to expand binomials:

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n}b^n$$

Ex 2.

Expand $(x+2)^4$

Ex 3.

Expand $(2x-y)^5$

In general, the $(r+1)$st term of the expansion of $(a+b)^n$ is $\binom{n}{r}a^{n-r}b^r$

Ex 4.

Find the fourth term in the expansion of $(3x+y)^9$

Practice

1. Evaluate the following:

a) $\binom{7}{3}$

b) $\binom{100}{98}$

c) $\binom{12}{0}$

d) $\binom{11}{11}$

2. Expand $(x^2 + y)^4$

3. Expand $(x - 3y)^5$

4. Find the sixth term in the expansion of $(x^3 + y^2)^8$

Q: A man worked at a high security institution. The man tried to log into his computer and the computer denied the password. He then remembered that the passwords to the computers were reset every month for security reasons. He called his boss for his new password. The man said, "Boss, my old password is out of date." The boss said, "Yes, it is. The new password is different, but if you listen closely you will be able to figure out the new one. Your new password has the same amount of letters as the old one, and four of the letters are the same." The man then logged into his computer with no trouble. What was the new password? What was his old?