Sequences and Summation Notation

A _______________ is an ordered list. ex: $1, 3, 5, 7, 9, \ldots$

The 1, 3, 5, 7, 9, etc. are called __________. So, the fourth term in our example is _____.

We can think of sequences as functions that take natural #'s as input. Input 1 to get the first term of the sequence, input 2 to get the second term, and so on…
ex: $f(n) = 2n - 1$
The list $f(1), f(2), f(3), f(4), f(5), \ldots$ becomes $1, 3, 5, 7, 9, \ldots$

A sequence can be **finite** or **infinite**.

ex: $2, 4, 6, 8, 10$ is ____________.

ex: $2, 4, 6, 8, 10, \ldots$ is ____________.

Rather than using $f(n)$, mathematicians usually use _____ for sequences (where n is a natural #).

So, the terms of a sequence called a_n are $a_1, a_2, a_3, a_4, \ldots$

Ex 1.
Write the first three terms of the sequence $a_n = \frac{n+1}{n+2}$

Ex 2.
Write the first four terms of the sequence $a_n = (-1)^{n+1} \cdot n^2$.

Factorial Notation

$\boldsymbol{n}! =$ ____

$\mathbf{0}! =$ ______

Ex 3.
$5! =$

$2 \cdot 5! =$

Ex 4.

Write the first three terms of the sequence $a_n = \frac{2^n}{(n+1)!}$

Summation Notation

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

Ex 5.

Expand and evaluate each sum.

$$\sum_{i=1}^{6} 2i^2$$

$$\sum_{k=3}^{5} (2^k - 3)$$

$$\sum_{i=1}^{5} 4$$

Ex 6.

Express each sum using summation notation. Use 1 as the lower limit of summation and i for the index of summation.

$1^2 + 2^2 + 3^2 + \cdots + 9^2$

$\frac{1}{3} + \frac{2}{4} + \frac{3}{5} + \cdots + \frac{16}{18}$

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^{n-1}}$

Practice

1. Write the first four terms of each sequence.

a) $a_n = (-1)^{n+1}(n+4)$

b) $a_n = \frac{(n+1)!}{n^2}$

2. Find each sum.

a) $\sum_{i=1}^{5} i^3$

b) $\sum_{k=2}^{4} (k-3)(k+2)$

3. Express each sum using summation notation. Use 1 as the lower limit of summation and i for the index of summation.

a) $5 + 5^2 + 5^3 + \cdots + 5^{12}$

b) $\frac{1}{9} + \frac{2}{9^2} + \frac{3}{9^3} + \cdots + \frac{n}{9^n}$

Q: What turns everything around, but does not move?