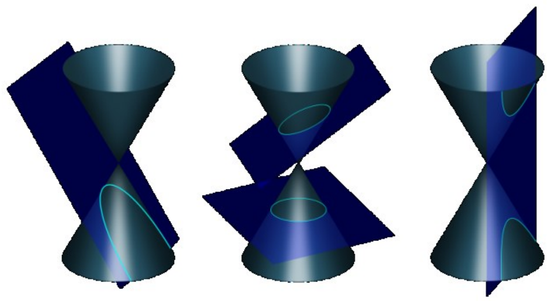
Ellipse, Hyperbola, and Parabola



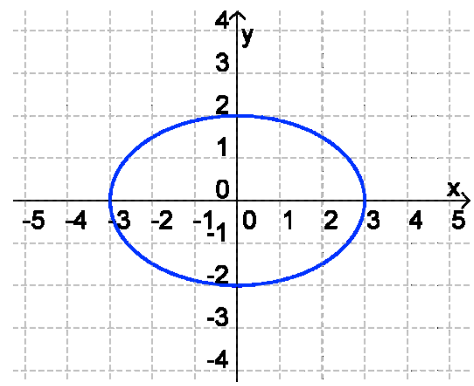
Intersections of a plane with a right circular cone are called ________________________.

Here are the conic sections we'll look at:
- Circle
- Ellipse
- Hyperbola
- Parabola

Ellipse

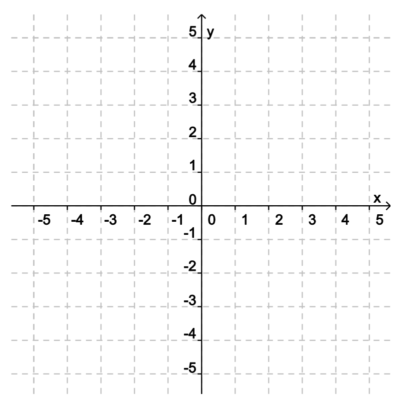
The equation for an ellipse centered at the origin is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



What are the x-intercepts?

What are the y-intercepts?

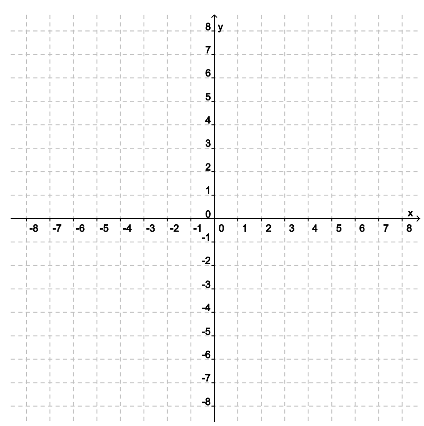


Ex 1.

Graph $\frac{x^2}{4} + \frac{y^2}{16} = 1$

Here's the standard form of a "shifted" ellipse, with center (h, k):

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



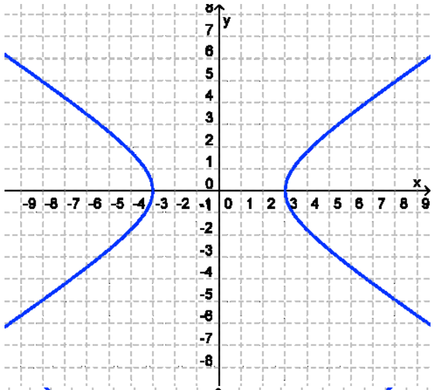
Ex 2.

Graph $\frac{(x-3)^2}{4} + \frac{(y+1)^2}{16} = 1$

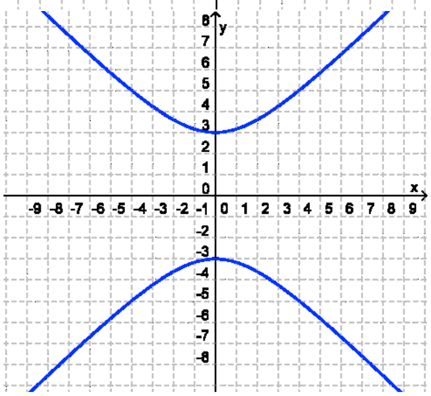
Hyperbola

The equations for a hyperbola centered at the origin are:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



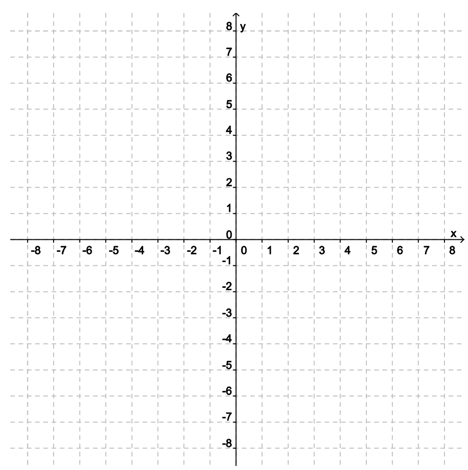
$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$



To graph hyperbolas, use a "box" to help draw the asymptotes…

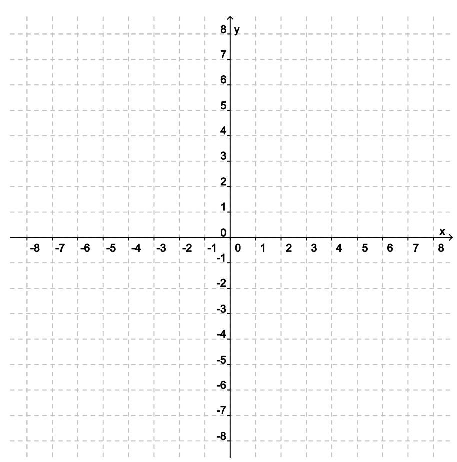
Ex 3.

Graph $\frac{x^2}{25} - \frac{y^2}{16} = 1$



Ex 4.

Graph $\frac{y^2}{9} - \frac{x^2}{4} = 1$



Parabola (Horizontal)

We already looked at vertical parabolas, where the equations looked like:

$y = ax^2 + bx + c$ and $y = a(x-h)^2 + k$

But parabolas can be horizontal, too. What do the equations look like?

$\boldsymbol{x = ay^2 + by + c}$

$\boldsymbol{x = a(y-k)^2 + h}$ (vertex at (h, k); if $a > 0$, opens right; if $a < 0$ opens left)

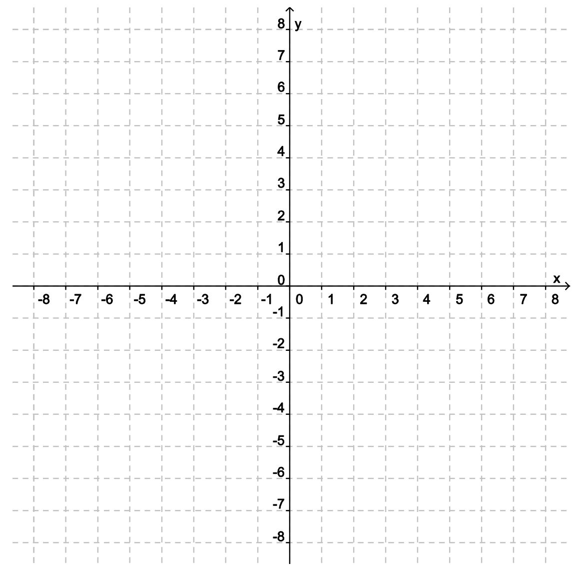
Ex 5.

Let's look at $x = -(y-2)^2 + 1$

1. Does this parabola open right or left? _________

2. What's the vertex? __________

3. Find the x-intercept.



4. Find the y-intercepts.

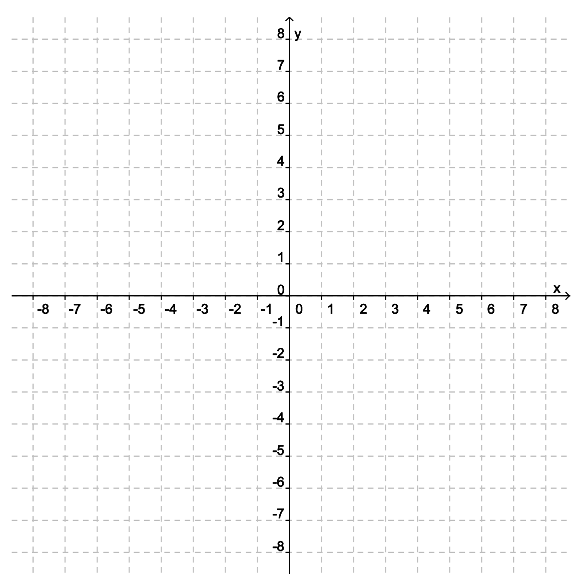
5. Graph it.

Ex 6.

Let's look at $x = y^2 + 2y - 3$

1. Does this parabola open right or left? _________

2. Find the vertex.



3. Find the x-intercept.

4. Find the y-intercepts.

5. Graph it!

Identifying Conic Sections by Their Equations

If only one variable squared, it's a _______________. (ex: $x = y^2 + 3y + 2$, or $y = 2x^2 - 4x + 1$)

Otherwise, move the x^2 and y^2 terms to one side, and look at their coefficients.

If same, it's a _____________. (ex: $4x^2 + 4y^2 = 9$)

If different coefficients, but same sign, it's an _______________. (ex: $4x^2 + 9y^2 = 36, \frac{y^2}{25} + x^2 = 1$)

If different coefficients, and opposite signs, it's a _____________. (ex: $4x^2 - 9y^2 = 36, \frac{y^2}{16} - \frac{x^2}{9} = 1$)

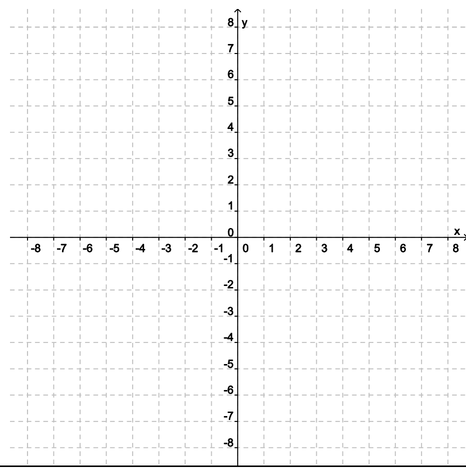
Ex 7.

Fill in the table.

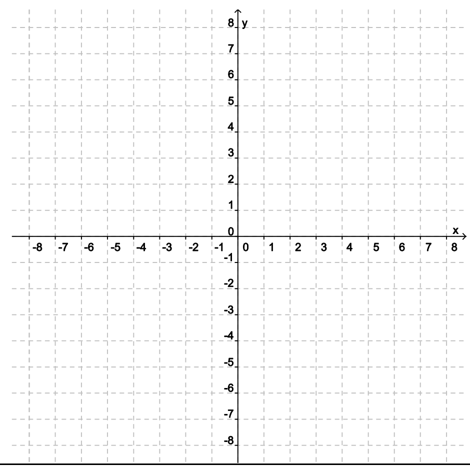
Equation	**Conic Section**
$x^2 = 4y^2 + 16$	
$x^2 = 16 - 4y^2$	
$4x^2 = 16 - 4y^2$	
$x = -4y^2 + 16y$	
$4y^2 - 16 = -x^2$	
$9y^2 = 4x^2 - 36$	

Practice

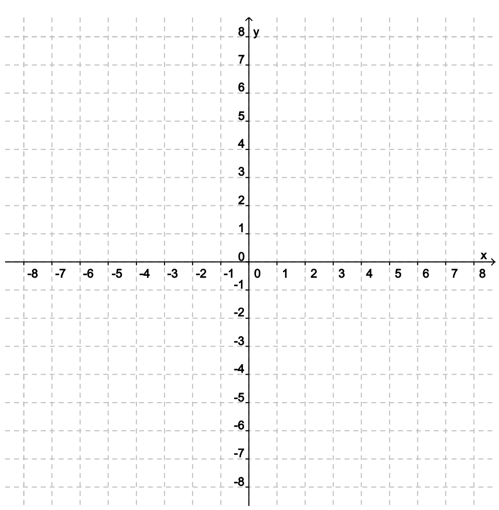
1. Identify the following conic section and graph.
(Hint: Divide both sides by 100 first.)
$4x^2 + 25y^2 = 100$



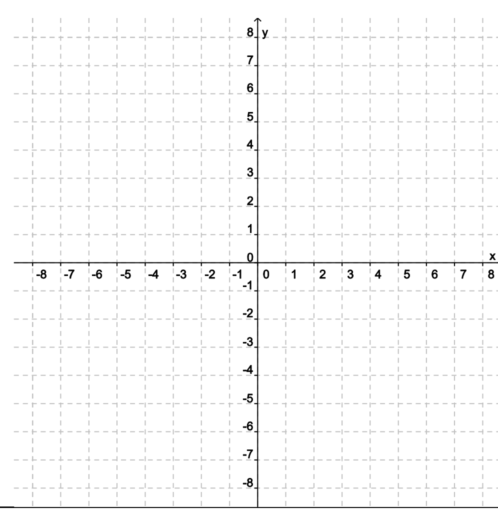
2. Identify the following conic section and graph.
(Hint: Rewrite x^2 as $\frac{x^2}{1}$)
$\frac{y^2}{9} - x^2 = 1$



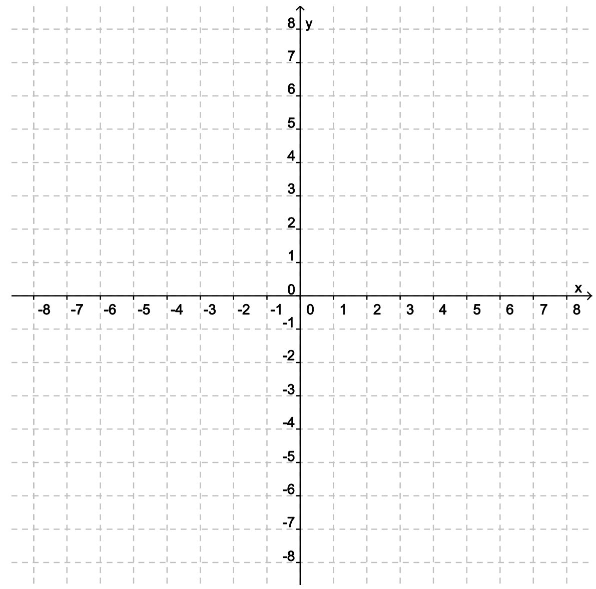
3. Identify the following conic section and graph.
$\frac{(x-3)^2}{9} + \frac{(y+1)^2}{16} = 1$



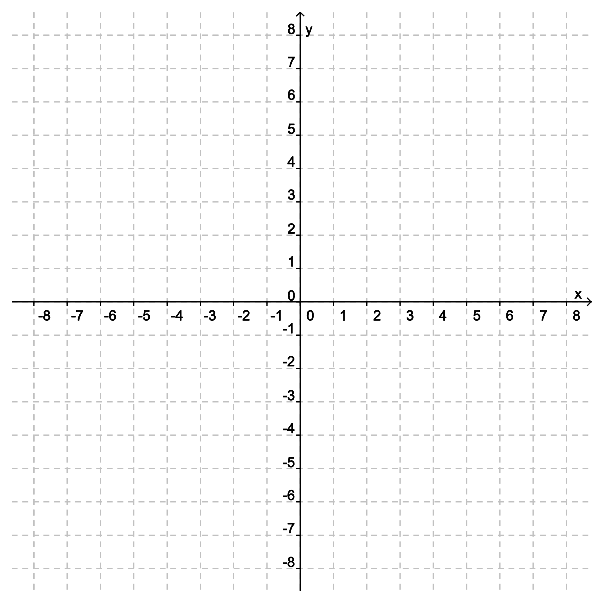
4. Identify the following conic section and graph.
(Hint: Divide both sides by 144 first.)
$16x^2 - 9y^2 = 144$



5. Consider the parabola $x = 3(y-1)^2 - 3$. Find the vertex, x-intercept, and y-intercepts. Then graph it.



6. Consider the parabola $x = -y^2 - 2y + 3$. Find the vertex, x-intercept, and y-intercepts. Then graph it.



Q: A boy was at a carnival and went to a booth where a man said to the boy, "If I write your exact weight on this piece of paper then you have to give me $50, but if I cannot, I will pay you $50." The boy looked around and saw no scale so he agreed, thinking no matter what the man writes he'll just say he weighs more or less. In the end the boy ended up paying the man $50. How did the man win the bet?