Systems of Linear Equations in Three Variables

Here's an example of a **linear equation in three variables**: $2x - 3y + z = 4$

Solutions

Ex 1.

Show that the ordered triple $(1, -1, 2)$ is a solution of the system:

$x - y + 2z = 6$

$3x + 2y - z = -1$

$-2x + 5y + z = -5$

Solving Systems with Three Variables

Goal: Take 2 different pairs of equations and eliminate the same variable from both pairs.

Ex 2.

Solve the system:

$2x + y - 3z = -5$

$x - 2y + 2z = 3$

$3x + 5y + z = 16$

Ex 3.

Solve the system:

$x - 3z = 5$

$4x + 2y + 3z = 11$

$-2x + 4y - z = 9$

Inconsistent and Dependent Systems

Ex 4.

Try solving the system:

$2x - y + 3z = 4$

$2x + 3y - z = 5$

$-4x + 2y - 6z = 1$

What happens and what does it mean?

Ex 5.

Now try solving the system:

$-x + y + z = 1$

$2x - 2y - 2z = -2$

$x + 2y - 3z = 4$

What happens and what does it mean?

Summary:

For systems of equations…

…if get a *false* statement (like $0 = 9$), then system is **inconsistent** (i.e. no solution).

…if get a *true* statement (like $0 = 0$), then system is **dependent** (i.e. infinitely many solutions).

Practice

1. Determine if the given ordered triple is a solution of the system.

$(-1, 3, 2)$

$x - 2z = -5$

$y - 3z = -3$

$2x - z = -4$

2. Solve the system. If the system is inconsistent or dependent, state so.

$2x + y - 2z = -1$

$3x - 3y - z = 5$

$x - 2y + 3z = 6$

3. Solve the system. If the system is inconsistent or dependent, state so.

$3x + 4y + 5z = 8$

$x - 2y + 3z = -6$

$2x - 4y + 6z = 8$

Q: What five-letter word becomes shorter when you add two letters to it?