Systems of Linear Equations in Two Variables

Suppose we have two linear equations:

$x + y = 3$

$x - y = -1$

Together, they make a _____ (also called a _____).

Solutions must satisfy *all* equations.

For example, $(1,2)$ is a solution to the above system since it satisfies *both* equations.

Ex 1.

Determine if $(1,1)$ is a solution to the following system:
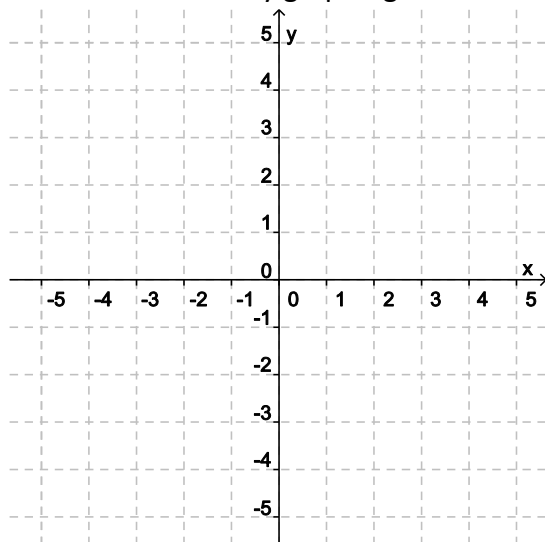
$-2x + 3y = 1$

$4x - 3y = 7$

Is $(4,3)$ a solution?

Graphing Systems of Linear Equations

Besides $(1, 2)$, are there any other solutions to the following system?

$x + y = 3$

$x - y = -1$

We can show this by graphing the two lines.

Substitution Method

Ex 2.

Solve by the substitution method:

$y = -3x + 1$

$4x - y = 2$

How to solve a system using the substitution method:

1. Solve one equation for x or y
2. Substitute the expression we get for x or y into the other equation
3. This makes an equation with one variable that we can solve
4. After that, we can find the other variable

Ex 3.

Solve by the substitution method:

$3x + 2y = 4$

$2x + y = 1$

Addition Method

Another way to solve linear systems.

Ex 4.

Solve by the addition method:

$3x + 4y = 2$
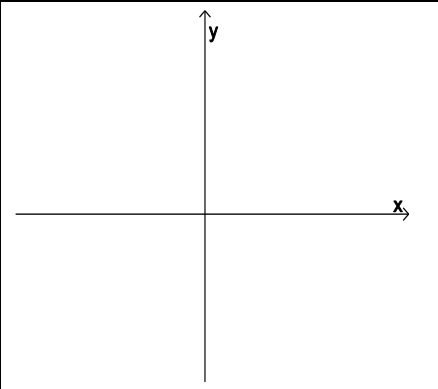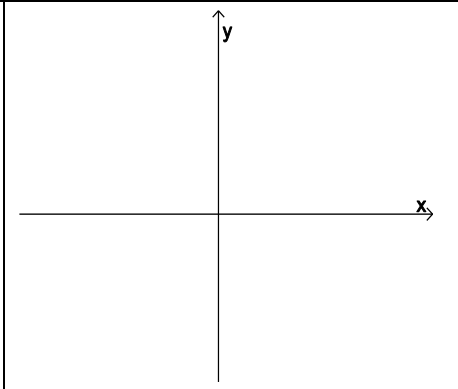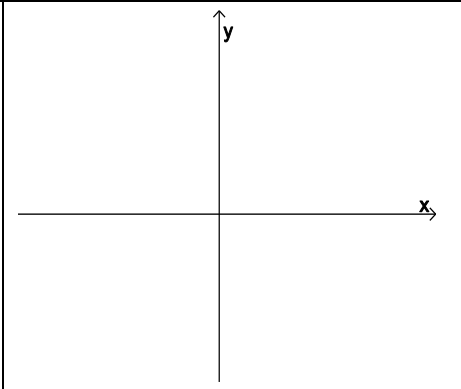
$2x + 5y = -1$

Ex 5.

Solve by the addition method (Hint: clear the fractions first!):

$\frac{7}{2}x + \frac{5}{2}y = -4$

$3x + \frac{2}{3}y = 1$

No Solution or Infinitely Many Solutions

Exactly one solution	No solution	Infinitely many solutions

Ex 6.

Solve the system:

$5x - 2y = 4$

$-10x + 4y = 7$

Ex 7.

Solve the system:

$x = 4y - 8$

$5x - 20y = -40$

Practice

1. Solve by the substitution method.

   $x = 3y + 8$

   $2x - y = 6$

2. Solve by the addition method.

   $x + 2y = -1$

   $2x - y = 3$

3. Solve the system (if it is inconsistent or dependent, say so).

   $y = 2x + 4$

   $y = 2x - 1$

4. Solve the system (if it is inconsistent or dependent, say so).

   $3x - 3y = 6$

   $2x = 2y + 4$

5. (Review) Evaluate: a) $\frac{0}{5}$ b) $-\frac{11}{0}$ c) -3^4 d) $(-2)^4$ e) $5 - 2(4 - 1)$

6. (Review) Simplify: a) $7x^4 \cdot (-2x^3)$ b) $(-3x^{-4})^2$ c) $(6x^3y^4z^{10})^0$

7. (Review) Solve: a) $11x - 4(2x - 3) = 18 + 5(x + 2)$ b) $4x + 7 = 7(x + 1) - 3x$

Q: What are two things people never eat before breakfast?