

## Functions

### Relations

The number of Drug Law Violations per year at Mt. SAC is given by the following table:

| Year | # of Drug Law Violations |
|------|--------------------------|
| 2007 | 3                        |
| 2008 | 4                        |
| 2009 | 2                        |
| 2010 | 4                        |

(Source: <http://www.mtsac.edu/safety/stats/>)

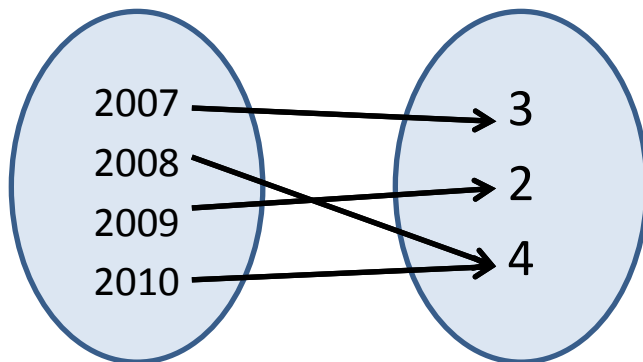
We can write this as a set of ordered pairs (called a \_\_\_\_\_):

$\{(2007, 3), (2008, 4), (2009, 2), (2010, 4)\}$

The set of all the first components is called the \_\_\_\_\_. Above, it is \_\_\_\_\_.

The set of all the second components is called the \_\_\_\_\_. Above, it is \_\_\_\_\_.

Relations can be visualized like this (called an “arrow diagram”):



#### Ex 1.

Find the domain and range of the relation  $\{(3, 5), (-2, 1), (3, 7), (0, 5)\}$ .

### Functions

A relation in which each member of the domain corresponds to exactly one member of the range is called a \_\_\_\_\_. (That is, **each input** has only **one output**.)

ex:  $\{(2007, 3), (2008, 4), (2009, 2), (2010, 4)\}$  is a function since each input has only one output.

ex:  $\{(3, 5), (-2, 1), (3, 7), (0, 5)\}$  is not a function since 3 has two outputs: \_\_\_ and \_\_\_.

## Functions as Equations

For this class, we'll focus on functions in the form of equations. For example,  $y = x^2 - 3x + 2$

We often give functions names (like  $f, g, h, F, G, H$ ) and use special notation to define them.

For example,  $f(x) = x^2 - 3x + 2$ .

$f(x)$  is read “ $f$  of  $x$ ”.

$f(x)$  represents the value of the function at  $x$  (that is, the output of  $f$ ).

For example,  $f(2) = (2)^2 - 3(2) + 2 = 0$ , that is “ $f$  of 2 is 0”. So, if you input 2, the output is 0.

### Ex 2.

Find  $h(-2)$  for the function  $h(x) = 2x^2 + 3x - 1$ .

### Ex 3.

Find  $F(a + h)$  for  $F(x) = 5x + 7$ .

### Ex 4.

Find  $f(71)$  for  $f(x) = 46$ .

## Functions as Tables

### Ex 5.

| $x$ | $f(x)$ |
|-----|--------|
| -2  | 4      |
| -1  | 1      |
| 0   | 0      |
| 1   | 1      |
| 2   | 4      |

What are the domain and range of the table to the left?

$f(-1) =$

Find  $x$  such that  $f(x) = 4$ .

### Interval Notation

Suppose you wanted to write “the set of all real numbers between 3 and 5, including 3, but not 5.”

That is, the set of all real numbers  $x$ , such that  $3 \leq x < 5$ .

Here’s how to write it using **interval notation**: \_\_\_\_\_

| Interval Notation   | Inequality        | Graph |
|---------------------|-------------------|-------|
| $(a, b)$            | $a < x < b$       |       |
| $[a, b]$            | $a \leq x \leq b$ |       |
| $[a, b)$            | $a \leq x < b$    |       |
| $(a, b]$            | $a < x \leq b$    |       |
| $(a, \infty)$       | $x > a$           |       |
| $[a, \infty)$       | $x \geq a$        |       |
| $(-\infty, b)$      | $x < b$           |       |
| $(-\infty, b]$      | $x \leq b$        |       |
| $(-\infty, \infty)$ | $\mathbb{R}$      |       |

#### Ex 6.

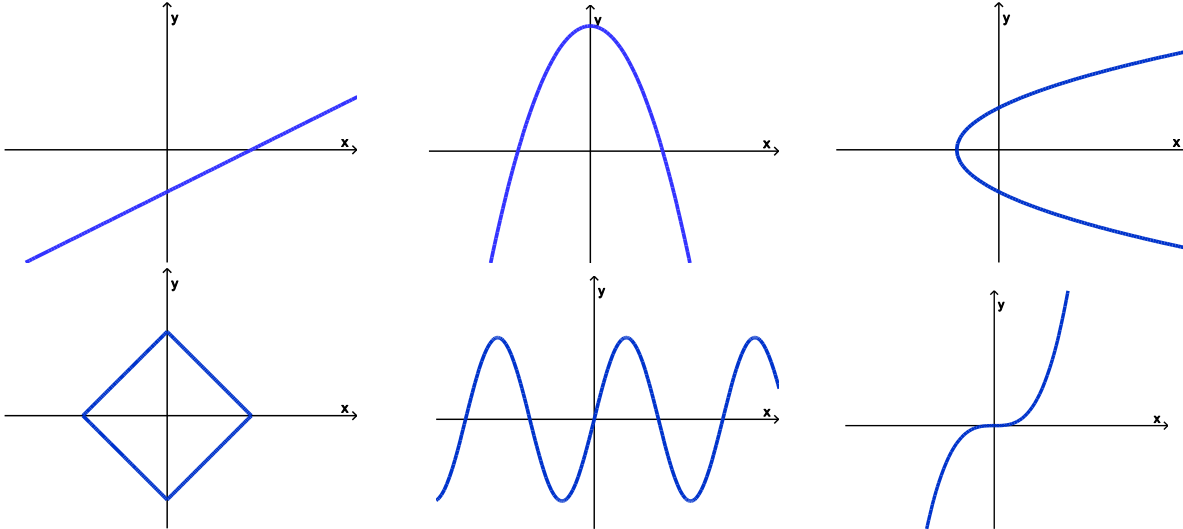
Fill in the table:

| Interval Notation | Inequality      | Graph |
|-------------------|-----------------|-------|
|                   | $-3 \leq x < 1$ |       |
| $(-2, 3]$         |                 |       |
|                   | $x \geq 2$      |       |
| $[2.5, 4]$        |                 |       |
| $(-\infty, -1)$   |                 |       |

If you plot all of a function’s ordered pairs, then you create the \_\_\_\_\_ of the function.  
 To test if a graph is a function we can use the \_\_\_\_\_. If any vertical line intersects a graph in two or more points, then the graph does not represent a function.

**Ex 7.**

Which of the following graphs are functions?

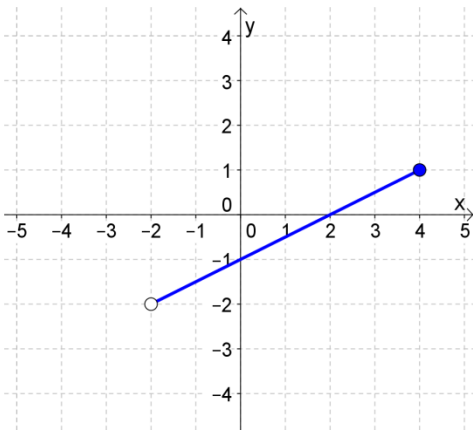


The \_\_\_\_\_ of a function is the set of all possible \_\_\_\_\_.

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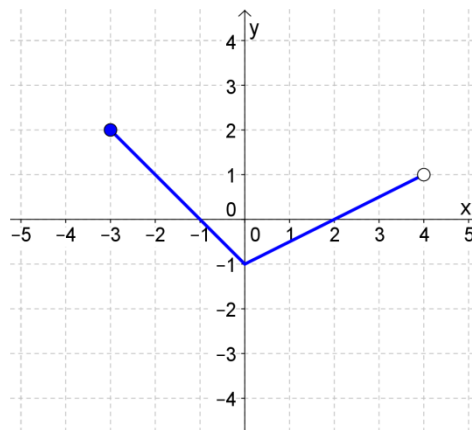
**Ex 8.**

Use the graph of each function to find its domain and range.



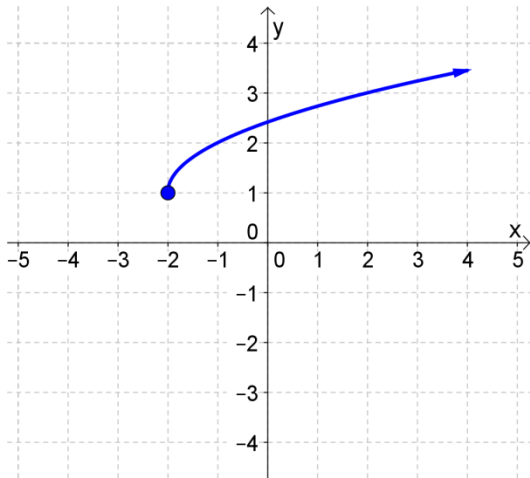
Domain \_\_\_\_\_

Range \_\_\_\_\_



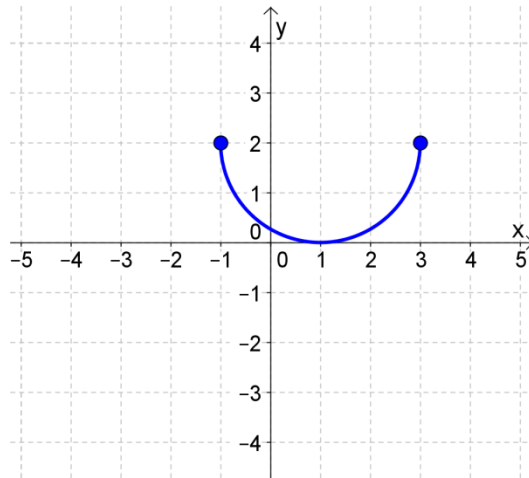
Domain \_\_\_\_\_

Range \_\_\_\_\_



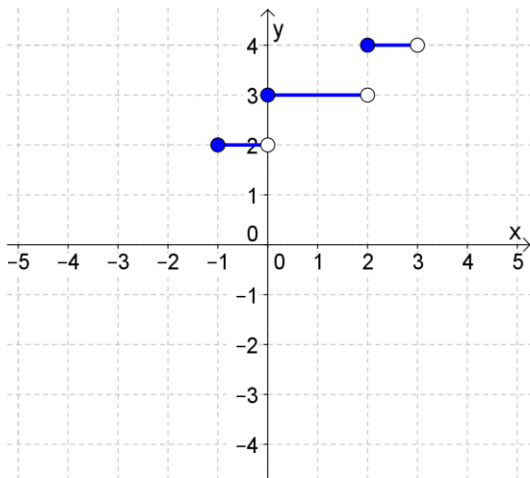
Domain \_\_\_\_\_

Range \_\_\_\_\_



Domain \_\_\_\_\_

Range \_\_\_\_\_



Domain \_\_\_\_\_

Range \_\_\_\_\_

**Summary**

Here is the same function represented in different ways.

| Set of Ordered Pairs               | Table  | Graph | Arrow Diagram |   |   |   |    |   |   |   |   |  |  |
|------------------------------------|--|-------|---------------|---|---|---|----|---|---|---|---|--|--|
| $\{(1,3), (2, -1), (3,0), (4,3)\}$ | <table border="1"> <thead> <tr> <th><math>x</math></th> <th><math>f(x)</math></th> </tr> </thead> <tbody> <tr> <td>1</td> <td>3</td> </tr> <tr> <td>2</td> <td>-1</td> </tr> <tr> <td>3</td> <td>0</td> </tr> <tr> <td>4</td> <td>3</td> </tr> </tbody> </table> | $x$   | $f(x)$        | 1 | 3 | 2 | -1 | 3 | 0 | 4 | 3 |  |  |
| $x$                                | $f(x)$   |       |               |   |   |   |    |   |   |   |   |  |  |
| 1                                  | 3  |       |               |   |   |   |    |   |   |   |   |  |  |
| 2                                  | -1   |       |               |   |   |   |    |   |   |   |   |  |  |
| 3                                  | 0  |       |               |   |   |   |    |   |   |   |   |  |  |
| 4                                  | 3  |       |               |   |   |   |    |   |   |   |   |  |  |

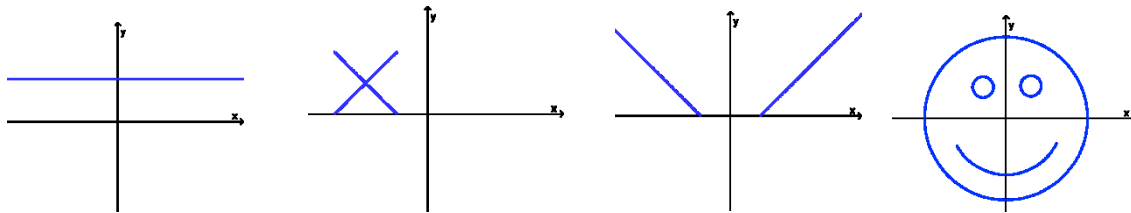
The domain of the above function is \_\_\_\_\_.

The range of the above function is \_\_\_\_\_.

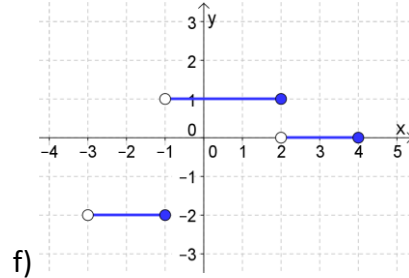
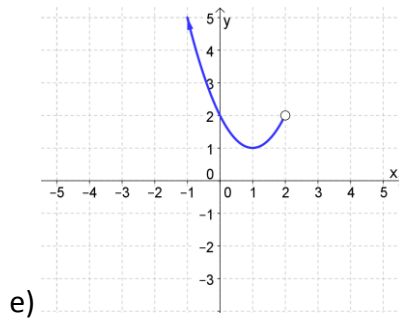
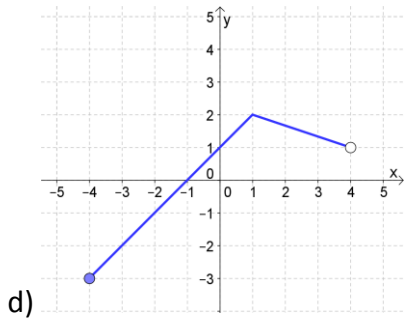
**Practice**

- Find the domain and range of the relation  $\{(4, 1), (5, 1), (6, 1)\}$ . Also, is the relation a function?
- For the function  $f(x) = 4x - 3$ , find  $f(-2)$ ,  $f\left(\frac{3}{4}\right)$ , and  $f(a + 2)$ . Be sure to simplify your results.
- For the function  $g(x) = 2x^2 - 3x + 1$ , find  $g(-1)$  and  $g(2x)$ . Be sure to simplify your results.
- Use the table to the right to answer the following questions.
  - What are the domain and range of  $f$ ?
  - Find  $f(3)$  and  $f(0)$ .
  - Find  $x$  such that  $f(x) = 12$ .
- Which of the following graphs are functions?

| $x$ | $f(x)$ |
|-----|--------|
| -5  | 3      |
| -3  | 12     |
| 0   | 2      |
| 3   | 7      |
| 5   | 12     |



- Use the graph of each function to find its domain and range.



Q: What can you catch, but not throw?