Using Rational Numbers as Exponents

Let's define what it means to have a rational # as an exponent: $a^{\frac{1}{n}} = $ _____

Ex 1.

$81^{\frac{1}{2}} = $ $\qquad\qquad\qquad\qquad (-64)^{1/3} = $

Note: We can write $a^{2/3}$ in a couple of different ways:

$$a^{\frac{2}{3}} = \left(a^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{a}\right)^2 \qquad \textbf{or} \qquad a^{\frac{2}{3}} = (a^2)^{\frac{1}{3}} = \sqrt[3]{a^2}$$

Radical Notation	Rational Exponent
$\sqrt[7]{x}$	
$\sqrt{x}$	
$\sqrt[3]{x^5}$	
$\sqrt{x^3}$	
	$x^{\frac{1}{8}}$
	$x^{\frac{1}{2}}$
	$x^{\frac{5}{6}}$
	$x^{\frac{7}{2}}$

Ex 2.

Simplify by first writing in radical form.

$8^{\frac{4}{3}} = $

$-81^{\frac{3}{4}} = $

Ex 3.

Rewrite with rational exponents:

$\sqrt[3]{5^4} = $ $\qquad\qquad\qquad \left(\sqrt[4]{11}\right)^9 = $

Ex 4.

Evaluate.

$100^{-\frac{1}{2}} = $

$32^{-\frac{3}{5}} = $

Note: The same rules that you learned before apply when working with rational exponents.

$$a^m \cdot a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n} \qquad (a^m)^n = a^{mn} \qquad (ab)^n = a^n b^n \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \qquad a^{-n} = \frac{1}{a^n} \qquad a^0 = 1$$

Ex 5.

Simplify. Write your answers in exponential form with only positive exponents. Assume all variables represent positive numbers.

$7^{\frac{1}{2}} \cdot 7^{\frac{1}{3}} = $

$\dfrac{5^{1/4}}{5^{3/4}} = $

$$\left(9^{1/4}\right)^2 =$$

$$\left(\frac{9}{4}\right)^{5/2} =$$

$$\frac{2^{1/2} \cdot 2^{-1}}{2^{-3/2}} =$$

$$\left(x^2 y^{1/2}\right)^4 =$$

$$\frac{x^{2/3} \cdot x^{-1/3}}{x^{5/3}} =$$

Practice

1. Simplify by first writing in radical form.

    a) $25^{\frac{3}{2}}$

    b) $(-32)^{1/5}$

    c) $27^{-\frac{2}{3}}$

2. Rewrite each expression with rational exponents.

    a) $\sqrt{17}$

    b) $\sqrt[3]{17}$

    c) $\sqrt[7]{x^4}$

3. Simplify. Write your answers in exponential form with only positive exponents. Assume all
    variables represent positive numbers.

    a) $2^{\frac{2}{5}} \cdot 2^{\frac{3}{5}}$

    b) $\left(32^{\frac{2}{3}}\right)^{\frac{3}{5}}$

    c) $\dfrac{x^{1/4}}{x^{-3/5}}$

    d) $\left(y^{-\frac{3}{4}}\right)^{\frac{1}{6}}$

    e) $\left(x^8 y^{2/3}\right)^{1/4}$

Q: What are the next two letters in the following series and why?

W A T N T L I T F S _ _