

Solving Linear Inequalities

Interval Notation

Suppose you wanted to write “the set of all real numbers between 3 and 5, including 3, but not 5.”

One way we can write this is using a **three-part inequality**: $3 \leq x < 5$.

Here’s how to write it using **interval notation**: _____

Interval Notation	Inequality	Graph
(a, b)	$a < x < b$	
$[a, b]$	$a \leq x \leq b$	
$[a, b)$	$a \leq x < b$	
$(a, b]$	$a < x \leq b$	
(a, ∞)	$x > a$	
$[a, \infty)$	$x \geq a$	
$(-\infty, b)$	$x < b$	
$(-\infty, b]$	$x \leq b$	
$(-\infty, \infty)$	\mathbb{R}	

Ex 1.

Fill in the table:

Interval Notation	Inequality	Graph
	$-3 \leq x < 1$	
$(-2, 3]$		
	$x \geq 2$	
$[2.5, 4]$		
$(-\infty, -1)$		

Here's an example of a **linear inequality in one variable**: $3x - 2 \leq 1$

Is 0 a solution?

Is 1 a solution?

Is 2 a solution?

The solution set in interval notation is: _____

Ex 2.

Solve and graph the solution set of $2x + 3 < -5$.

Note: Solving inequalities is similar to solving equations, with one *important* difference:

If _____ both sides by a _____, then _____ inequality symbol.

Why? Here's an example:

$$\begin{aligned}1 &< 2 \\(-1) \cdot 1 &> (-1) \cdot 2 \\-1 &> -2\end{aligned}$$

Ex 3.

Solve $-3x + 4 \leq 10$, and graph the solution set.

Ex 4.

Solve $4(x - 1) - 3x > -15 - (2x + 1)$, and graph the solution set.

Note: When solving a three-part inequality (like in the next example), operations must be done to all three parts.

Ex 5.

Solve $2 \leq 3x - 1 \leq 8$, and graph the solution set.

Note: $1 < x < 4$ and $4 > x > 1$ mean the same thing, but it is more standard to write $1 < x < 4$.

Practice

1. Fill in the table:

Interval Notation	Inequality	Graph
	$-1 \leq x < 2$	
$[-3, -1)$		
$(0, \infty)$		
$[-\frac{1}{3}, 2]$		
	$x \leq -2$	

2. Solve $5(x - 3) - 7x \geq 4(x - 3) + 9$, and graph the solution set. (Write your solution in interval notation.)

3. Solve $-3 < \frac{3}{5}x - 1 \leq 8$, and graph the solution set.

Q: What goes around the world but stays in a corner?