

Exponents

Product Rule: $a^m \cdot a^n = \underline{\hspace{2cm}}$

Quotient Rule: $\frac{a^m}{a^n} = \underline{\hspace{2cm}}$ ($a \neq 0$)

Power Rule 1: $(a^m)^n = \underline{\hspace{2cm}}$

Power Rule 2: $(ab)^n = \underline{\hspace{2cm}}$

Power Rule 3: $\left(\frac{a}{b}\right)^n = \underline{\hspace{2cm}}$ ($b \neq 0$)

To see why these rules are true, expand the exponents and simplify. For example,

$$b^4 \cdot b^3 =$$

$$\frac{b^5}{b^3} =$$

$$(b^2)^3 =$$

$$(ab)^3 =$$

$$\left(\frac{a}{b}\right)^4 =$$

Here are a few rules to handle zero and negative exponents (note: $a \neq 0$)...

Zero-Exponent Rule: $a^0 = \underline{\hspace{2cm}}$

Negative-Exponent Rule: $a^{-n} = \underline{\hspace{2cm}}$

Negative Exponents in Denominators: $\frac{1}{a^{-n}} = \underline{\hspace{2cm}}$

Note how these make the Quotient Rule (and the Product Rule) consistent:

$$\frac{b^3}{b^5} = \frac{b \cdot b \cdot b}{b \cdot b \cdot b \cdot b \cdot b} = \frac{1}{b^2}$$

$$\frac{b^3}{b^5} = b^{3-5} = b^{-2}$$

$$\frac{b^4}{b^4} = \frac{b \cdot b \cdot b \cdot b}{b \cdot b \cdot b \cdot b} = 1$$

$$\frac{b^4}{b^4} = b^{4-4} = b^0$$

Okay, last two helpful rules...I promise!

Negative-to-positive: $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$ and $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

Examples: $\frac{2^{-4}}{5^{-3}} = \frac{5^3}{2^4}$ and $\left(\frac{4}{7}\right)^{-2} = \left(\frac{7}{4}\right)^2$

Ex 1.

$$(2x^2y^3)(4x^5y^2) =$$

Ex 2.

$$\frac{6a^{20}b^{12}}{3a^{15}b^3} =$$

Ex 3.

$$(x^{100})^5 =$$

Ex 4.

$$(-2x^3y^7)^3 =$$

Ex 5.

$$(2x^{-3})^5 =$$

Ex 6.

$$\left(\frac{a^2}{3}\right)^3 =$$

Simplifying Exponential Expressions

(use each step if needed)

1. Remove parentheses using $(ab)^n = a^n b^n$ or $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

2. Simplify powers of powers using $(a^m)^n = a^{mn}$

3. Use $a^m \cdot a^n = a^{m+n}$ or $\frac{a^m}{a^n} = a^{m-n}$

4. Use $a^0 = 1$, and make sure all exponents are positive using $a^{-n} = \frac{1}{a^n}$ or $\frac{1}{a^{-n}} = a^n$

Ex 7.

$$\left(\frac{2x^3}{y^{-4}}\right)^5 =$$

Ex 8.

$$(-3x^{-6}y)(-2x^3y^4)^2 =$$

Ex 9.

$$\left(\frac{10x^3y^5}{5x^6y^{-2}}\right)^2 =$$

Ex 10.

$$\left(\frac{x^{-2}y^7}{2}\right)^{-5} =$$

Practice

1. Simplify.

a) $(4x^5y^6)(20x^7y^4)$ b) $\frac{36x^{12}y^4}{4xy^2}$ c) $17y^0$ d) x^3y^{-4} e) $\frac{x^{-3}}{y^{-7}}$ f) $(3x^2y)^4$ g) $\left(-\frac{2x}{y}\right)^5$

h) $\frac{15x^2}{3x^{11}}$ i) $\frac{24x^2y^{13}}{-2x^5y^{-2}}$ j) $\left(\frac{4a^{-5}b^3}{12a^3b^{-5}}\right)^0$ k) $\left(\frac{x^6y^{-7}}{2}\right)^{-3}$

Q: What has four wheels and flies?