Solving Systems of Linear Equations

Suppose we have two linear equations:

$x + y = 3$

$x - y = -1$

Together, they make a _____ (also called a _____).

Solutions must satisfy *all* equations.

For example, $(1,2)$ is a solution to the above system since it satisfies *both* equations.

Ex 1.

Determine if $(1,1)$ is a solution to the following system:

$-2x + 3y = 1$

$4x - 3y = 7$

Is $(4,3)$ a solution?

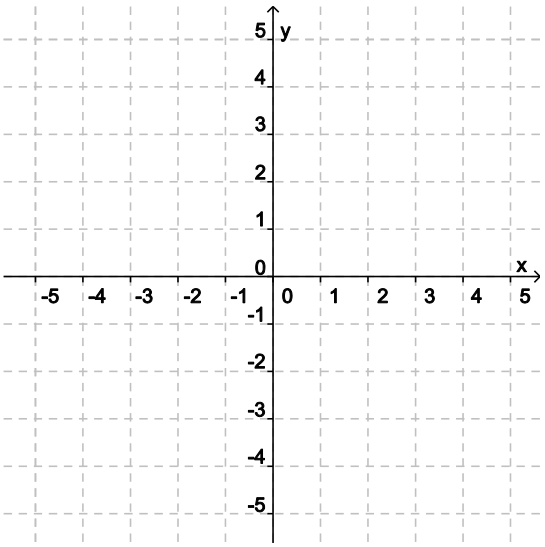
Graphing Systems of Linear Equations

Besides $(1, 2)$, are there any other solutions to the following system?

$x + y = 3$

$x - y = -1$

We can show this by graphing the two lines.



Substitution Method

Ex 2.

Solve by the substitution method:

$y = -3x + 1$

$4x - y = 2$

How to solve a system using the substitution method:

1. Solve one equation for x or y
2. Substitute the expression we get for x or y into the other equation
3. This makes an equation with one variable that we can solve
4. After that, we can find the other variable

Ex 3.

Solve by the substitution method:

$3x + 2y = 4$

$2x + y = 1$

Elimination Method

Another way to solve linear systems.

Ex 4.

Solve by the elimination method:

$3x + 4y = 2$

$2x + 5y = -1$

How to solve a system using the elimination method:

1. Write equations in form $Ax + By = C$
2. Use multiplication to make coefficients of x or y add up to zero
3. Add equations (after which one variable should be eliminated)
4. This makes an equation with one variable that we can solve
5. After that, we can find the other variable

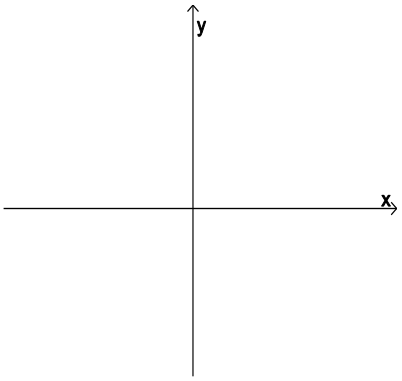
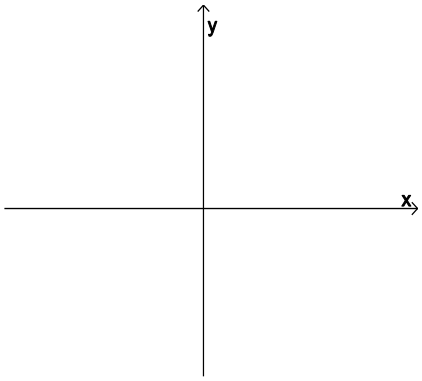
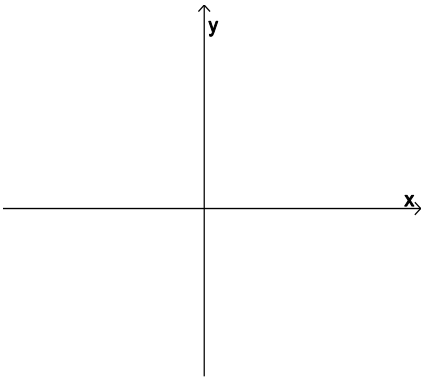
Ex 5.

Solve by the elimination method (Hint: clear the fractions first!):

$\frac{7}{2}x + \frac{5}{2}y = -4$

$3x + \frac{2}{3}y = 1$

No Solution or Infinitely Many Solutions

Ex 6.

Solve the system:

$5x - 2y = 4$

$-10x + 4y = 7$

Ex 7.

Solve the system:

$x = 4y - 8$

$5x - 20y = -40$

Summary

When solving a system of linear equations, if you…

…get a **false** statement, then the system is **inconsistent**, the solution set is ∅, and the lines are **parallel**.

…get a **true** statement, then the system is **dependent**, the solution set is $\{(x, y)|\textbf{write either equation here}\}$, and the lines are **identical**.

Practice

1. Decide whether $(4, -1)$ is a solution of the system.

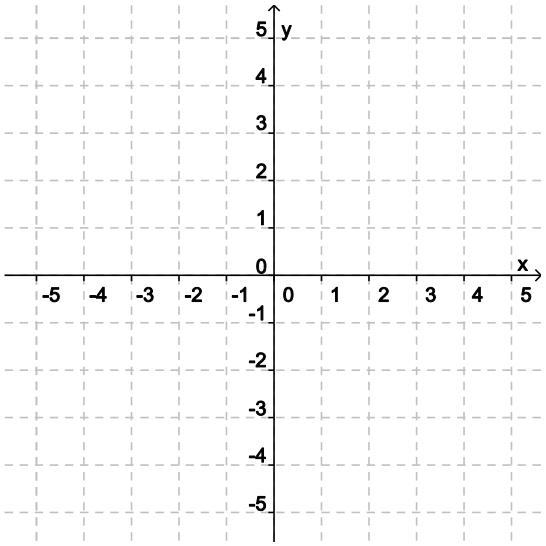
$5x + 6y = 14$

$2x + 5y = 3$

2. Solve by graphing.

$x - 2y = 4$

$2x + y = 3$



3. Solve by the substitution method.

$x = 3y + 8$

$2x - y = 6$

4. Solve by the elimination method.

$x + 2y = -1$
$2x - y = 3$

5. Solve the system (if it is inconsistent or dependent, say so).

$y = 2x + 4$
$y = 2x - 1$

6. Solve the system (if it is inconsistent or dependent, say so).

$3x - 3y = 6$
$2x = 2y + 4$

Q: What gets wet when drying?