Exponent Rules; Multiplying Polynomials

Multiplying Monomials

Notice that: $2^3 \cdot 2^4 = (2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2) = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7$

So, we could write: $2^3 \cdot 2^4 = 2^{3+4} = 2^7$

This rule works when multiplying exponentials with the same base. So, in general:

$\boldsymbol{n^a \cdot n^b = n^{a+b}}$

Ex 1.

Multiply.

$(x^5)(x^2) =$

$(2x^3)(6x^8) =$

$-4y^2(3y^3)(2y) =$

Monomials Raised to a Power

Notice that: $(2^3)^2 = 2^3 \cdot 2^3 = 2^{3+3} = 2^6$

Also, $(2^3)^5 = 2^3 \cdot 2^3 \cdot 2^3 \cdot 2^3 \cdot 2^3 = 2^{3+3+3+3+3} = 2^{15}$

Since repeated addition is multiplication, we could write:

$(2^3)^5 = 2^{3 \cdot 5} = 2^{15}$

In general:

$\boldsymbol{(n^a)^b = n^{a \cdot b}}$

Ex 2.

Simplify: $(t^4)^3$

How about $(2x^3)^4$?

$$\begin{aligned}(2x^3)^4 &= 2x^3 \cdot 2x^3 \cdot 2x^3 \cdot 2x^3 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot x^3 \cdot x^3 \cdot x^3 \cdot x^3 \\ &= 2^4(x^3)^4 \\ &= 16x^{12}\end{aligned}$$

So, when you have a monomial raised to a power:

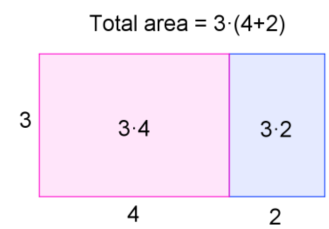
1. Evaluate ________________ raised to the power.
2. ________________ each variable's _______________ by the power.

Ex 3.
Simplify: $(-2x^4)^3$

Ex 4.
Simplify: $(5y^9)^2$

Multiplying a Monomial and a Polynomial

What property allows you to take the following step?
$3(4+2) = 3 \cdot 4 + 3 \cdot 2$



We can use the same property with algebraic expressions. For example,
$3(4x+2) = 3 \cdot 4x + 3 \cdot 2$

Ex 5.
Multiply: $-2x^2(2x^3 - 3x + 5)$

Multiplying Polynomials

Note: The distributive property works from the right as well. For example, $(4+2) \cdot 3 = 4 \cdot 3 + 2 \cdot 3$
Let's try multiplying the following: $(x+3)(x+2)$
Distributing the $(x+3)$, we get: $(x+3) \cdot x + (x+3) \cdot 2$
Distributing again, we get: $x^2 + 3x + 2x + 6$
Simplifying, we get: $x^2 + 5x + 6$

So, to multiply polynomials: multiply **each** term of the **second** polynomial by **each** term of the **first**.

Ex 6.
Multiply: $(2x + 2)(x - 5)$

Ex 7.
Multiply: $(x - 3)(2x^2 - 3x - 5)$

Note: When multiplying two binomials, the mnemonic **FOIL** is sometimes helpful.
FOIL stands for **F**irst **O**uter **I**nner **L**ast.
For example, $(3x - 1)(2x + 4) = \underbrace{3x \cdot 2x}_{\text{First}} + \underbrace{3x \cdot 4}_{\text{Outer}} - \underbrace{1 \cdot 2x}_{\text{Inner}} - \underbrace{1 \cdot 4}_{\text{Last}}$

Binomials that differ only in the sign separating the terms are called ______________________.
ex: $x + 3$ and $x - 3$
ex: $-2y - 3$ and $-2y + 3$

Ex 8.
What's the conjugate of $3x + 4$?

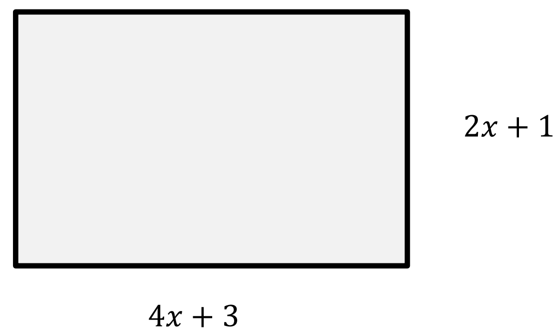
Ex 9.
Multiply: $(3x + 4)(3x - 4)$. What happens?

In general, $(\boldsymbol{a} + \boldsymbol{b})(\boldsymbol{a} - \boldsymbol{b}) =$ ______________

Ex 10.
Multiply $(2x + 6)(2x - 6)$ using the above rule.

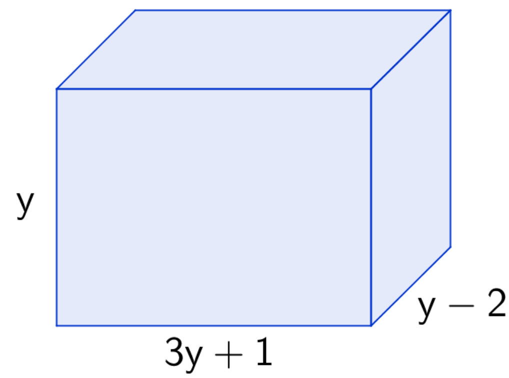
Ex 11.

Write an expression for the area of the following rectangle, and simplify it.



Ex 12.

Write an expression for the volume of the following box, and simplify it.



Summary

$n^a \cdot n^b = n^{a+b}$

$(n^a)^b = n^{a \cdot b}$

Monomial to a power: evaluate coefficient to power, multiply variable exponents by power.

Multiplying polynomials: multiply **each** term of the **second** polynomial by **each** term of the **first**.

$(a + b)(a - b) = a^2 - b^2$