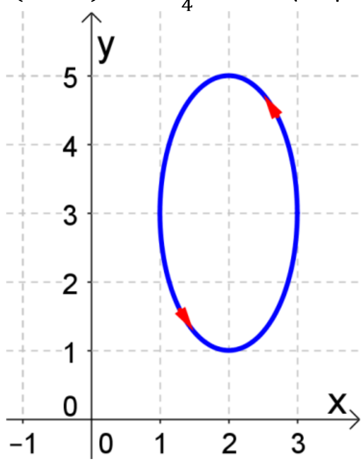
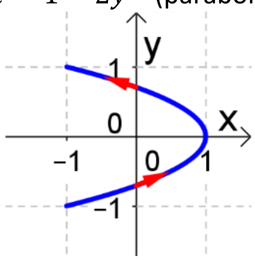
Math 181 - Test #3 Review Exercise Answers

1.

a. $(x-2)^2 + \frac{(y-3)^2}{4} = 1$ (ellipse)



b. $x = 1 - 2y^2$ (parabola opening left)



2. $y - \frac{2}{3} = \frac{3}{2}(x-3)$ (or $y = \frac{3}{2}x - \frac{23}{6}$)

3. $\frac{1}{(1-\sin t)^2}$ (Note: $\frac{dy}{dx} = \frac{\cos t}{1-\sin t}$)

4. $\frac{2}{27}\left(37\sqrt{37} - 10\sqrt{10}\right)$ (Hint: do some algebra and then let $u = \sqrt{t}$ to solve integral)

5. $\frac{2\sqrt{2}}{5}\pi(2e^{\pi} + 1)$

6.

a. $\frac{9\pi}{2}$ (Setup: $4 \cdot \int_0^{\pi/2} \frac{1}{2}(3\sin 2\theta)^2 d\theta$)

b. $18\sqrt{3} - 4\pi$ (Setup: $\int_{-\pi/3}^{\pi/3} \frac{1}{2}(4 + 4\cos\theta)^2 d\theta - \int_{-\pi/3}^{\pi/3} \frac{1}{2}(6)^2 d\theta$)

c. $\frac{5\pi}{4}$ (Setup: $2 \cdot \left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}(1+\sin\theta)^2 d\theta - \int_0^{\frac{\pi}{2}} \frac{1}{2}(\sin\theta)^2 d\theta\right)$)

d. $\frac{5\pi}{4}$ (Setup: $2 \cdot \left(\int_0^{\frac{\pi}{3}} \frac{1}{2}(1+\cos\theta)^2 d\theta + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{2}(3\cos\theta)^2 d\theta\right)$)

e. $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$ (Setup: $2 \cdot \left(\int_0^{\frac{\pi}{6}} \frac{1}{2}(2\sin\theta)^2 d\theta + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2}(1)^2 d\theta\right)$)

f. $4\pi - 6\sqrt{3}$ (Setup: $\int_{7\pi/6}^{11\pi/6} \frac{1}{2}(2 + 4\sin\theta)^2 d\theta$)

7. $\frac{\sqrt{5}}{2}(e^4 - 1)$ (Setup: $\int_0^2 \sqrt{(e^{2\theta})^2 + (2e^{2\theta})^2}\, d\theta$)

8. 2π (Setup: $\int_0^{\pi\sqrt{2}} \sqrt{\left(\sqrt{1+\sin 2\theta}\right)^2 + \left(\frac{\cos 2\theta}{\sqrt{1+\sin 2\theta}}\right)^2}\, d\theta$)

9. $-\sqrt{3}$

10. Horizontal: $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$; Vertical: $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ (Note: you'll have to take the limit as $\theta \to \frac{\pi}{2}$ of $\frac{dy}{dx}$ to determine what happens at $\frac{\pi}{2}$)

11.
- a. converges to $-\frac{\pi}{2}$
- b. converges to 0 (Hint: use Squeeze Theorem)
- c. converges to $\sqrt{2}$
- d. diverges
- e. converges to 1
- f. converges to 0
- g. converges to 0
- h. converges to 0
- i. converges to 1
- j. converges to 0 (separate, then Squeeze Theorem)
- k. converges to 0 (geometric sequence)

12. 5

13. $1+\sqrt{2}$

14.
- a. divergent (Limit Comparison Test with $\frac{1}{n}$)
- b. divergent (Test for Divergence)
- c. convergent (Alternating Series Test)
- d. convergent (Integral Test)
- e. converges to $\frac{64}{125}$ (geometric series with $a = \frac{64}{625}$ and $r = \frac{4}{5}$)
- f. converges to $-\frac{2}{33}$ (geometric series with $a = -\frac{2}{27}$ and $r = -\frac{2}{9}$)
- g. divergent (geometric series with $r = \frac{\pi}{3} > 1$)
- h. divergent (Test for Divergence)
- i. convergent (Comparison Test with $\frac{2}{n^{3/2}}$)
- j. convergent (Comparison Test with $\frac{5}{n^2}$)
- k. convergent (Alternating Series Test)
- l. convergent (Alternating Series Test)

15.
- a. $-\frac{\pi}{4}$
- b. $\frac{3}{2}$

16. $\frac{3979}{550}$

17.

a. At most $\frac{1}{484}$ (≈ 0.0021)
b. At least 112 terms
c. 0.05184824862

18.

a. At most $\frac{1}{\ln 6}$ (≈ 0.55811063)
b. At least 485,165,196 terms (that is, $> e^{20}$)
c. 2.1120790183 (Note that $s_6 \approx 1.5760745338$ represents the sum of the terms $n = 2$ through $n = 6$, which is the first five terms. Your integrals should give you $\frac{1}{\ln 7}$ and $\frac{1}{\ln 6}$.)

19. -0.2834 (Need to add first 4 terms to get this, so that $|s - s_4| < b_5 = \frac{1}{3^5 5!} = \frac{1}{29160} < 0.00005$.)

20. 0.9721 (Need to add first 7 terms to get this, so that $|s - s_7| < b_8 = \frac{1}{8^5} = \frac{1}{32768} < 0.00005$.)

21.

a. Converges absolutely (Root Test)
b. Converges absolutely (Ratio Test)
c. Diverges (Root Test)
d. Converges conditionally (For $\sum \left|\frac{(-1)^{n+1}n}{n^2+2}\right|$, Comparison Test with $\sum \frac{1}{n}$. For $\sum \frac{(-1)^{n+1}n}{n^2+2}$, Alternating Series Test.)
e. Converges absolutely (Ratio Test)