

### Test #3 (Part 2, Scientific Calculator Okay)

Math 181, Prof. Beydler

Name: \_\_\_\_\_

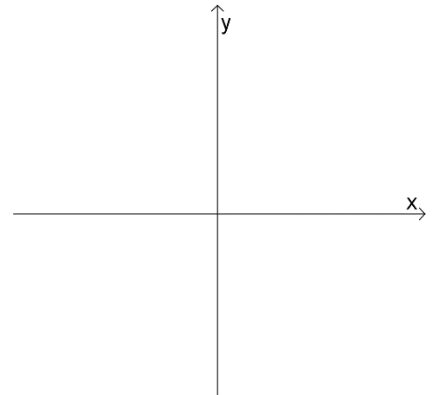
Wednesday, November 28, 2018

**Directions:** Show all work. No books or notes. A **scientific calculator** is allowed. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. Good luck!

1. (3 points) Given the following parametric equations/intervals of a particle in the  $xy$ -plane, find the related Cartesian equation and graph it. Then, indicate the portion of the graph traced by the particle and the direction of motion.

$$x = 3 \sin t, \quad y = 2 \cos t, \quad 0 \leq t \leq \pi$$

Cartesian equation of curve: \_\_\_\_\_



2. (4 points) Find the length of the curve  $x = \cos^3 t, y = \sin^3 t, 0 \leq t \leq \frac{\pi}{2}$

Answer: \_\_\_\_\_

3. Set up but do not evaluate (an) integral(s) to find the area of the region...

a. (4 points) ...inside  $r = 3 \cos \theta$  and outside  $r = 1 + \cos \theta$ .

Answer: \_\_\_\_\_

b. (4 points) ...inside both  $r = 4 \sin \theta$  and  $r = 2$ .

Answer: \_\_\_\_\_

4. (4 points) Find the length of the curve  $r = \theta^2$ ,  $0 \leq \theta \leq 2\pi$ . Leave your answer in exact form.

Answer: \_\_\_\_\_

5. (6 points) Find the values of  $\theta$  in  $[0, 2\pi)$  where the tangent line of  $r = 1 + \cos \theta$  is horizontal or vertical.

Horizontal when  $\theta =$  \_\_\_\_\_

Vertical when  $\theta =$  \_\_\_\_\_

6. (3 points) Assume that the following sequence converges and find its limit.

$$a_1 = 1, \quad a_{n+1} = \sqrt{2a_n + 3}$$

Answer: \_\_\_\_\_

7. Determine whether each series is convergent or divergent. If it converges, find its sum.

a. (3 points)  $\sum_{n=0}^{\infty} \frac{(-2)^n}{e^{n+1}}$

Convergent or divergent (circle one)

Sum (if it converges): \_\_\_\_\_

b. (4 points)  $\sum_{n=1}^{\infty} \frac{2}{(n+1)(n+3)}$

Convergent or divergent (circle one)

Sum (if it converges): \_\_\_\_\_

8. Use  $\sum_{n=1}^{\infty} \frac{1}{n^2}$  to answer the parts below.

a. (3 points) How many terms are needed to make sure that the sum is accurate to within 0.0005?

Answer: \_\_\_\_\_

b. (3 points) Use  $\boxed{3}$  with  $n = 4$  to give an estimate of the series' sum that's better than  $s_4$ .

Here's formula  $\boxed{3}$ :  $s_n + \int_{n+1}^{\infty} f(x) dx \leq \sum a_n \leq s_n + \int_n^{\infty} f(x) dx$

Answer: \_\_\_\_\_

9. (2 points) Find the sum of the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n n!}$  correct to 4 decimal places.

Answer: \_\_\_\_\_

Here are the two integrals I promised in case you need them:

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C$$
$$\int \csc x \, dx = -\ln |\csc x + \cot x| + C$$