_____ / 72 total points (both parts)

Test #3 (Part 1, No Calculator) Name: ________________________________

Math 181, Prof. Beydler Wednesday, November 28, 2018

Directions: Show all work. No calculator, books, or notes. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. When you're finished with Part 1, please turn it in, take a bathroom break, get your calculator out, and start Part 2. Good luck!

1. Determine whether each sequence converges or diverges. If it converges, find the limit.

a. (2 points) $a_n = \cos\left(\ln\left(\frac{n-1}{n+2}\right)\right)$

$\frac{n-1}{n+2} \to 1$, $\ln\left(\frac{n-1}{n+2}\right) \to 0$, $\cos\left(\ln\left(\frac{n-1}{n+2}\right)\right) \to 1$

Converges or diverges (circle one)

Limit (if convergent): 1

b. (3 points) $a_n = (n-1)^{1/n}$ (be sure to show your reasoning)

$$\lim_{n\to\infty} \ln(n-1)^{1/n}$$

$$= \lim_{n\to\infty} \frac{\ln(n-1)}{n}$$

$$= \lim_{n\to\infty} \frac{\left(\frac{1}{n-1}\right)}{1}$$

$$= 0$$

So,

$$\lim_{n\to\infty} (n-1)^{1/n}$$

$$= \lim_{n\to\infty} e^{\ln(n-1)^{1/n}}$$

$$= e^0 = 1$$

Converges or diverges (circle one)

Limit (if convergent): 1

2. Determine whether each series is convergent or divergent. Be sure to state any test that you use and show your reasoning. If you use the Integral Test or Alternating Series Test, be sure to state the conditions of the test and (if necessary) show why the conditions are met.

a. (3 points) $\sum_{n=1}^{\infty} \frac{3-\sin n}{n^2}$

$$\frac{3-\sin n}{n^2} \le \frac{4}{n^2}$$

Convergent or divergent (circle one)

Test used: Comparison Test

So since $\sum \frac{4}{n^2}$ converges (p-series with $p=2$), $\sum \frac{3-\sin n}{n^2}$ converges by comparison.

b. (4 points) $\sum_{n=0}^{\infty} \frac{3n+1}{\sqrt{2n^4+5n+3}}$

$$\frac{3n+1}{\sqrt{2n^4+5n+3}} \sim \frac{3n}{\sqrt{2n^4}} = \frac{3n}{\sqrt{2}n^2} = \frac{3}{\sqrt{2}n}$$

Convergent or divergent (circle one)

Test used: Limit Comparison Test

$$\lim_{n\to\infty} \frac{\left(\frac{3n+1}{\sqrt{2n^4+5n+3}}\right)}{\left(\frac{1}{n}\right)} = \lim_{n\to\infty} \frac{(3n^2+n) \div n^2}{\left(\sqrt{2n^4+5n+3}\right) \div \sqrt{n^4}} = \lim_{n\to\infty} \frac{3+\frac{1}{n}}{\sqrt{2+\frac{5}{n^3}+\frac{3}{n^4}}} = \frac{3}{\sqrt{2}}$$

So since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, so does $\sum_{n=0}^{\infty} \frac{3n+1}{\sqrt{2n^4+5n+3}}$

c. (4 points) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$

$$b_n = \frac{n}{n^2+1}$$

Convergent or divergent (circle one)

Test used: Alternating Series Test

1. $\frac{n}{n^2+1} > 0$ for $n \geq 1$
2. $\frac{d}{dn}\left(\frac{n}{n^2+1}\right) = \frac{(n^2+1)(1)-(n)(2n)}{(n^2+1)^2} = \frac{1-n^2}{(n^2+1)^2}$ ← $1-n^2<0$, $n^2>1$, $n>1$

   Decreasing for $n \geq 2$
3. $\lim_{n\to\infty} \frac{n}{n^2+1} = \lim_{n\to\infty} \frac{\frac{1}{n}}{1+\frac{1}{n^2}} = 0$

d. (4 points) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

$\frac{1}{x \ln x}$ is continuous, positive, and decreasing for $x \geq 2$.

Convergent or divergent (circle one)

Test used: Integral Test

$$\int_2^{\infty} \frac{1}{x\ln x}\,dx = \lim_{t\to\infty} \int_2^t \frac{1}{x\ln x}\,dx$$

$u = \ln x$
$du = \frac{1}{x}\,dx$

$$= \lim_{t\to\infty} \int_{\ln 2}^{\ln t} \frac{1}{u}\,du$$

$$= \lim_{t\to\infty} [\ln u]_{\ln 2}^{\ln t}$$

$$= \lim_{t\to\infty} \left(\ln(\ln t) - \ln(\ln 2)\right)$$

$$= \infty$$

e. (3 points) $\sum_{n=1}^{\infty} \tan^{-1} n$

$$\lim_{n\to\infty} \tan^{-1} n = \frac{\pi}{2} \neq 0$$

Convergent or divergent (circle one)

Test used: Test for Divergence

3. Determine whether each series converges absolutely, converges conditionally, or diverges. Be sure to show your reasoning and state any test(s) used.

a. (3 points) $\sum_{n=1}^{\infty} \left(\frac{2+3n}{2n+1}\right)^{2n}$

Converges absolutely or converges conditionally or diverges (circle one)

Test(s) used: Root Test

$$\lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{2+3n}{2n+1}\right)^{2n}\right|}$$

$$= \lim_{n\to\infty} \left(\frac{2+3n}{2n+1}\right)^{2}$$

$$= \lim_{n\to\infty} \frac{9n^2+12n+4}{4n^2+4n+1}$$

$$= \frac{9}{4} > 1$$

b. (3 points) $\sum_{n=1}^{\infty} \frac{2^n(n+1)}{n!}$

Converges absolutely or converges conditionally or diverges (circle one)

Test(s) used: Ratio Test

$$\lim_{n\to\infty} \left| \frac{\cancelto{2}{2^{n+1}}(n+2)}{\cancelto{n+1}{(n+1)!}} \cdot \frac{\cancelto{1}{n!}}{\cancelto{1}{2^n}(n+1)} \right|$$

$$= \lim_{n\to\infty} \frac{2n+4}{n^2+2n+1}$$

$$= 0 < 1$$