Math 181 - Test #2 Review Exercise Answers

1.

a. $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{3}}\right)+C$

b. $\frac{4}{3}\left(\sqrt{x}+1\right)^{3/2}-4\sqrt{\sqrt{x}+1}+C$

c. $x-\frac{1}{2}\ln(1+e^{2x})+C$

d. $\frac{1}{2}x^2e^{x^2}-\frac{1}{2}e^{x^2}+C$

e. $\frac{1}{4}$ (that is, the integral converges to $\frac{1}{4}$)

f. $\frac{\pi}{3}$ (≈ 1.0472)

g. diverges

h. 0 (setup: $\lim\limits_{t\to 1^-}\int_0^t \frac{1}{(x-1)^{1/3}}\,dx+\lim\limits_{t\to 1^+}\int_t^2 \frac{1}{(x-1)^{1/3}}\,dx$)

i. $\frac{14\sqrt{2}}{3}$ (setup: $\lim\limits_{t\to 3^-}\int_1^t \frac{x}{\sqrt{3-x}}\,dx$)

2.

a. $\int_1^3 x\ln x\,dx\approx 2.966569,\quad |E_T|\le\frac{1}{24}=0.041\overline{6}$

b. $n=259$

c. $\int_1^3 x\ln x\,dx\approx 2.944021,\quad |E_S|\le\frac{1}{720}=0.0013\overline{8}$

d. $n=14$

e. $\int_1^3 x\ln x\,dx\approx 2.943755$

3.

a. Since $\frac{x^2-x-3}{\sqrt{x^8+x^5+2}}<\frac{x^2}{\sqrt{x^8}}=\frac{x^2}{x^4}=\frac{1}{x^2}$, and $\int_1^\infty \frac{1}{x^2}\,dx$ converges (since $p=2$), it follows by the Comparison Test that $\int_1^\infty \frac{x^2-x-3}{\sqrt{x^8+x^5+2}}\,dx$ converges.

b. Since $\frac{x^2+2}{(x-1)^3}>\frac{x^2}{x^3}=\frac{1}{x}$, and $\int_2^\infty \frac{1}{x}\,dx$ diverges (since $p=1$), it follows by the Comparison Test that $\int_2^\infty \frac{x^2+2}{(x-1)^3}\,dx$ diverges.

c. Since $\frac{\sin^2 x+x+x^5}{\sqrt{x^{12}-5x^7-23x}}>\frac{x^5}{\sqrt{x^{12}}}=\frac{x^5}{x^6}=\frac{1}{x}$, and $\int_3^\infty \frac{1}{x}\,dx$ diverges (since $p=1$), it follows by the Comparison Test that $\int_3^\infty \frac{\sin^2 x+x+x^5}{\sqrt{x^{12}-5x^7-23x}}\,dx$ diverges.

d. Since $\frac{\sqrt{x}-1}{x^3+3x^2}<\frac{\sqrt{x}}{x^3}=\frac{1}{x^{5/2}}$, and $\int_2^\infty \frac{1}{x^{5/2}}\,dx$ converges (since $p=\frac{5}{2}$), it follows by the Comparison Test that $\int_2^\infty \frac{\sqrt{x}-1}{x^3+3x^2}\,dx$ converges.

4.

a. $\ln\left(\frac{2+\sqrt{3}}{\sqrt{2}+1}\right)\approx 0.435584$

b. 45

5. $\frac{\pi\cdot 10^{3/2}-\pi}{27}\approx 3.5631$

6. $\frac{208\pi}{9}$

7. 4237 lb (here's a possible setup: $\int_{-4}^0 62.5(1-y)\cdot 2\sqrt{16-y^2}\,dy$)

8. 243911 N (here's a possible setup: $\int_0^4 1000\cdot 9.8\cdot(4-y)\left(\frac{12-2y}{3}\right)dy$)

9. $\left(2, \frac{8}{5}\right)$ (here are the setups: $\bar{x} = \frac{1}{A}\int_0^4 x(4x - x^2)\,dx = 2$; $\bar{y} = \frac{1}{A}\int_0^4 \left(\frac{4x-x^2}{2}\right)(4x - x^2)\,dx = \frac{8}{5}$; $A = \int_0^4 (4x - x^2)\,dx = \frac{32}{3}$; also, note that you can get $\bar{x}$ just from the symmetry of the region)

10. $\left(\frac{\pi}{2}, \frac{\pi}{8}\right)$ (here are the setups: $\bar{x} = \frac{1}{A}\int_0^\pi x \sin x\,dx = \frac{\pi}{2}$; $\bar{y} = \frac{1}{A}\int_0^\pi \left(\frac{\sin x}{2}\right)(\sin x)\,dx = \frac{\pi}{8}$; $A = \int_0^\pi \sin x\,dx = 2$; also, note that you can get $\bar{x}$ just from the symmetry of the region)

11. $\left(\frac{8}{3\pi}, 0\right)$ (here are the setups: $\bar{x} = \frac{1}{A}\int_{-2}^2 \left(\frac{\sqrt{4-y^2}}{2}\right)\left(\sqrt{4-y^2}\right)dy$; $\bar{y} = \frac{1}{A}\int_{-2}^2 y\sqrt{4-y^2}\,dy = 0$; $A = \int_{-2}^2 \sqrt{4-y^2}\,dy = 2\pi$; note that you can get $\bar{y}$ by symmetry, and you can get A by the area of half a circle: $\frac{1}{2}\pi(2)^2$)

12. $y' = Ae^x - 5Be^{-5x}$ and $y'' = Ae^x + 25Be^{-5x}$, so
$y'' + 4y' - 5y = Ae^x + 25Be^{-5x} + 4(Ae^x - 5Be^{-5x}) - 5(Ae^x + Be^{-5x})$
$= Ae^x + 4Ae^x - 5Ae^x + 25Be^{-5x} - 20Be^{-5x} - 5Be^{-5x}$
$= 0$

13. $y' = te^{-t} - e^{-t}$ and $y'' = -te^{-t} + e^{-t} + e^{-t} = -te^{-t} + 2e^{-t}$, so
$y'' + 2y' = -te^{-t} + 2e^{-t} + 2(te^{-t} - e^{-t}) = te^{-t}$.
Also, $y(0) = -(0)e^{-0} + 6 = 6$, and $y'(0) = 0e^{-0} - e^{-0} = -1$.

14.
   a. $T > T_S$
   b. $T < T_S$
   c. $T = T_S$

15.
   a. $y = \sin\left(\frac{1}{2}\ln(x^2 + 1) + C\right)$
   b. $y = -\ln(e^{-x} + C)$
   c. $y = 2 - Ce^{-x^2/2}$

16.
   a. $-\frac{2}{2x^2+2x-1}$
   b. $y = \pm\sqrt{e^{-2x}\left(-x^2 - x - \frac{1}{2}\right) + 1}$

17. $\frac{2}{3}x^2 + y^2 = C$

18. $y = \frac{C}{x}$

19. 7.7925% (the DE is $\frac{dA}{dt} = 2 - \frac{A}{10}$ and the solution to the DE is $A(t) = 20 - 12e^{-t/10}$, $A(10) = 15.585$ gallons of alcohol)