Math 181 - Test #2 Info and Review Exercises

Fall 2018, Prof. Beydler

Test Info

- Date: Wednesday, October 24, 2018
- Will cover sections 7.5, 7.7, 7.8, 8.1-8.3, 9.1, 9.3.
- You'll have the entire class to finish the test.
- This will be a 2-part test. Part 1 will be **no calculator**. Part 2 will be **scientific calculator only**.
- No notes, no books, no phones, no smart watches during the test.
- There will be a seating chart for the test.
- Where to get help as you're studying:
    - Office hours
    - TMARC, LAC, or other tutoring centers
    - E-mail me at dbeydler@mtsac.edu

Here are **some** of the formulas/concepts that you'll need to know:

Partial fraction decomposition

Linear factor $(x-r)$: $\frac{A}{x-r}$

Repeated linear factor $(x-r)^m$: $\frac{A_1}{x-r}+\frac{A_2}{(x-r)^2}+\cdots+\frac{A_m}{(x-r)^m}$

Irreducible quadratic factor x^2+px+q: $\frac{Ax+B}{x^2+px+q}$

Repeated irreducible quadratic factor $(x^2+px+q)^n$: $\frac{B_1x+C_1}{(x^2+px+q)}+\frac{B_2x+C_2}{(x^2+px+q)^2}+\cdots+\frac{B_nx+C_n}{(x^2+px+q)^n}$

Improper integrals (Type I)

$$\int_a^{\infty} f(x)\,dx = \lim_{t\to\infty}\int_a^t f(x)\,dx$$

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{t\to-\infty}\int_t^b f(x)\,dx$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_c^{\infty} f(x)\,dx$$

Improper integrals (Type II)

$$\int_a^b f(x)\,dx = \lim_{t\to a^+}\int_t^b f(x)\,dx \qquad \text{(discontinuity at } x=a)$$

$$\int_a^b f(x)\,dx = \lim_{t\to b^-}\int_a^t f(x)\,dx \qquad \text{(discontinuity at } x=b)$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx \qquad \text{(discontinuity at } x=c)$$

$\frac{1}{x^p}$ integrals

$\int_1^\infty \frac{1}{x^p}\,dx$ converges if $p > 1$ and diverges if $p \le 1$.

Comparison Test

Suppose f and g are continuous on $[a, \infty)$ and $0 \le f(x) \le g(x)$ in $[a, \infty)$.

If $\displaystyle\int_a^\infty g(x)\,dx$ converges, then $\displaystyle\int_a^\infty f(x)\,dx$ converges.

If $\displaystyle\int_a^\infty f(x)\,dx$ diverges, then $\displaystyle\int_a^\infty g(x)\,dx$ diverges.

Arc length

$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$ (or $L = \int_a^b \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$)

Surface area

$S = \int 2\pi r\,ds$ (often $S = \int_a^b 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$ or $S = \int_a^b 2\pi x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$)

Hydrostatic force

$P_i = \rho g d,\ F_i = P_i A_i$

Centroid (center of mass with uniform density)

$\overline{x} = \frac{1}{A}\int \tilde{x}\,dA$ $\overline{y} = \frac{1}{A}\int \tilde{y}\,dA$ $(\tilde{x}, \tilde{y})$ is the centroid of a thin strip, and $dA = (\text{length of strip}) \cdot dx$

I'll give you these formulas if you need them:

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

Trapezoidal Rule

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

$|E_T| \le \frac{M(b-a)^3}{12n^2}$ (M is any upper bound of $|f''|$ on $[a, b]$, and n is # of subintervals)

Simpson's Rule

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

$|E_S| \le \frac{M(b-a)^5}{180n^4}$ (M is any upper bound of $\left|f^{(4)}\right|$ on $[a, b]$, and n is # of subintervals)

Review Exercises

Note: If you write up solutions to all of the review exercises listed below, and hand them in at the test, you can earn up to 2% extra credit towards your test! It is important to understand that these review exercises are <u>not guaranteed to cover all</u> of the potential problems on the test. Please review the notes and homework problems to fully prepare for the test.

Types of problems that will appear on Part 1 are labeled **NC** (for **N**o **C**alculator).

1. Find the following integrals. If the integral diverges, write "diverges." (**NC**)

   a. $\int \frac{\sqrt{x}}{x^2+3x}\,dx$

   b. $\int \frac{1}{\sqrt{\sqrt{x}+1}}\,dx$

   c. $\int \frac{1}{1+e^{2x}}\,dx$

d. $\int x^3 e^{x^2}\, dx$

e. $\int_0^\infty x e^{-2x}\, dx$

f. $\int_{-\infty}^{\infty} \frac{dx}{x^2+9}$

g. $\int_{-\infty}^{0} \frac{x dx}{\sqrt{4-x}}$

h. $\int_{0}^{2} \frac{dx}{(x-1)^{1/3}}$

i. $\int_{1}^{3} \frac{x dx}{\sqrt{3-x}}$

2. Consider the integral $\int_1^3 x \ln x \, dx$.

a. Use the Trapezoidal Rule with $n = 4$ steps to approximate the integral, and then estimate the error in the approximation.

b. How large does n need to be to guarantee that the approximation from part (a) is accurate to within 0.00001?

c. Use Simpson's Rule with $n = 4$ steps to approximate the integral, and then estimate the error in the approximation.

d. How large does n need to be to guarantee that the approximation from part (c) is accurate to within 0.00001?

e. Evaluate the integral directly.

3. Show that each of the following integrals either converge or diverge using the Comparison Test. (**NC**)

a. $\int_1^\infty \frac{x^2-x-3}{\sqrt{x^8+x^5+2}}\,dx$

b. $\int_2^\infty \frac{x^2+2}{(x-1)^3}\,dx$

c. $\int_3^\infty \frac{\sin^2 x+x+x^5}{\sqrt{x^{12}-5x^7-23x}}\,dx$

d. $\int_2^\infty \frac{\sqrt{x}-1}{x^3+3x^2}\,dx$

4. Find the length of each of the following curves.

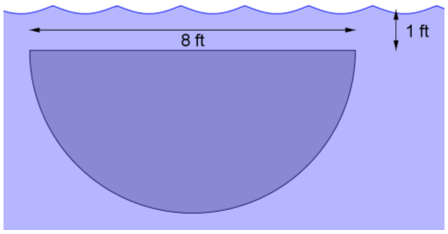
a. $y = \ln(\cos x)$ from $x = \frac{\pi}{4}$ to $x = \frac{\pi}{3}$

b. $y = \frac{2}{3}(x^2 + 1)^{3/2}$ from $x = 1$ to $x = 4$

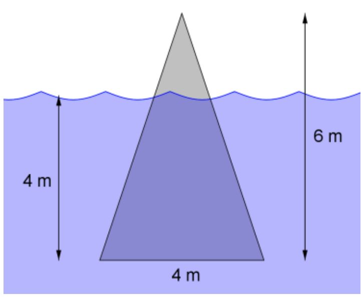
5. Find the area of the surface generated by revolving the curve $y = x^3$ from $x = 0$ to $x = 1$ about the x-axis.

6. Find the area of the surface generated by revolving the curve $y = \frac{x^3}{6} + \frac{1}{2x}$ from $x = 1$ to $x = 3$ about the x-axis.

7. A semicircular vertical plate is submerged in water as shown. Use a Riemann sum to approximate the hydrostatic force against one side of the plate. Then find the exact hydrostatic force against one side of the plate.



8. A vertical plate in the shape of an isosceles triangle is partially submerged in water as shown. Use a Riemann sum to approximate the hydrostatic force against one side of the plate. Then find the exact hydrostatic force against one side of the plate.



9. Find the centroid of the region enclosed by $y = 4x - x^2$ and $y = 0$.

10. Find the centroid of the region enclosed by $y = \sin x$, $x = 0$, $x = \pi$, and $y = 0$.

11. Find the centroid of the region enclosed by $x = \sqrt{4 - y^2}$ and $x = 0$.

12. Show that $y = Ae^{x} + Be^{-5x}$ is a solution to the following differential equation. (Note: A and B are constants.) (**NC**)
$y'' + 4y' - 5y = 0$

13. Show that $y = -te^{-t} + 6$ is a solution to the following initial-value problem. (Note: Here, you have to show that the DE and **both** initial conditions are satisfied.) (**NC**)
$y'' + 2y' = te^{-t},\quad y(0) = 6,\quad y'(0) = -1$

14. The following DE models the temperature $T(t)$ of an object: $\frac{dT}{dt} = -k(T - T_S)$. This is called Newton's Law of Cooling. T_S is the surrounding temperature, and k is a positive constant that depends on the type of object. (**NC**)
a. For what values of T will the temperature be decreasing?

b. For what values of T will the temperature be increasing?

c. For what value(s) of T will the temperature be constant? (In these cases, they're called equilibrium solutions.)

15. Find the general solution to each of the following differential equations. (**NC**)

a. $\frac{dy}{dx} = \frac{x\sqrt{1-y^2}}{x^2+1}$

b. $\frac{dy}{dx} = e^{y-x}$

c. $y' + xy = 2x$

16. Solve each initial value problem.

a. $y' = y^2 + 2xy^2, \quad y(0) = 2$

b. $ye^{2x}\frac{dy}{dx} = x^2, \quad y(0) = -\frac{1}{\sqrt{2}}$

17. Find the orthogonal trajectories of the family of curves $y^2 = kx^3$, where k is an arbitrary constant. (**NC**)

18. Find the orthogonal trajectories of the family of curves $x^2 - y^2 = k$, where k is an arbitrary constant. (**NC**)

19. A 200-gallon vat is full of a solution that's 4% alcohol. A solution with 10% alcohol starts pouring into the vat at 20 gal/min. Assume the solution in the vat is kept well-mixed and drains from the vat at 20 gal/min. Find the percentage of alcohol in the vat's solution after 10 minutes.