_____ / 54 total points (both parts)

Test #2 (Part 1, No Calculator)
Math 181, Prof. Beydler

Name: ________________________________

Wednesday, October 24, 2018

Directions: Show all work. No calculator, books, or notes. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. When you're finished with Part 1, please turn it in, take a bathroom break, get your calculator out, and start Part 2. Good luck!

1. Find the following integrals. If the integral diverges, write "diverges."

a. (5 points) $\int \sqrt{x}e^{\sqrt{x}}\,dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}dx$

$2u\,du = dx$

$= \int ue^u \cdot 2u\,du$

$= 2\int u^2e^u\,du$

$u^2 \quad e^u$

$2u \quad e^u$

$2 \quad e^u$

$0 \quad e^u$

$+ \quad - \quad +$

$= 2(u^2e^u - 2ue^u + 2e^u) + C$

Answer: $2xe^{\sqrt{x}} - 4\sqrt{x}e^{\sqrt{x}} + 4e^{\sqrt{x}} + C$

b. (4 points) $\int_2^\infty \frac{\ln x}{x}\,dx$

$= \lim_{t\to\infty} \int_2^t \frac{\ln x}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}dx$

$= \lim_{t\to\infty} \int_{\ln 2}^{\ln t} u\,du$

$= \lim_{t\to\infty} \left[\frac{u^2}{2}\right]_{\ln 2}^{\ln t}$

$= \lim_{t\to\infty} \left[\frac{(\ln t)^2}{2} - \frac{(\ln 2)^2}{2}\right]$

$= \infty$

Answer: diverges

c. (4 points) $\int_1^3 \frac{dx}{x-2}$

Answer: diverges

$$= \int_1^2 \frac{dx}{x-2} + \int_2^3 \frac{dx}{x-2}$$

$$= \lim_{t\to 2^-} \int_1^t \frac{dx}{x-2} + \lim_{t\to 2^+} \int_t^3 \frac{dx}{x-2}$$

$$= \lim_{t\to 2^-} \left[\ln|x-2|\right]_1^t + \lim_{t\to 2^+} \left[\ln|x-2|\right]_t^3$$

$$= \underbrace{\lim_{t\to 2^-} \left[\ln|t-2| - \cancel{\ln|1-2|}^{\,0}\right]}_{\to -\infty} + \underbrace{\lim_{t\to 2^+} \left[\cancel{\ln|3-2|}^{\,0} - \ln|t-2|\right]}_{\to \infty}$$

2. (2 points) Show that the following integral either converges or diverges using the Comparison Test.

$\int_2^\infty \frac{\sqrt{x^4+3x+1}}{2x^3-5} dx$

converges diverges (circle one)

$$\frac{\sqrt{x^4+3x+1}}{2x^3-5} > \frac{\sqrt{x^4}}{2x^3} = \frac{x^2}{2x^3} = \frac{1}{2x}$$

Since $\int_2^\infty \frac{1}{2x} dx$ diverges $(p=1)$, we have that

$\int_2^\infty \frac{\sqrt{x^4+3x+1}}{2x^3-5} dx$ diverges by Comparison. $\boxed{///}$

3. (4 points) Solve the following initial value problem. Be sure to solve explicitly for y.
$y' = xy^2 \sin 2x,\ y(0) = \frac{1}{2}$

$y = \dfrac{4}{2x\cos 2x - \sin 2x + 8}$

$$\int \frac{1}{y^2}\,dy = \int x \sin 2x\,dx$$

$$-\frac{1}{y} = -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x + C$$

$$-\frac{1}{y} = -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x + C$$

$$y = \frac{1}{\frac{1}{2}x\cos 2x - \frac{1}{4}\sin 2x + 2}$$

x	$+$	$\sin 2x$
1	$-$	$-\frac{1}{2}\cos 2x$
0		$-\frac{1}{4}\sin 2x$

Find C:

$$-\frac{1}{(\frac{1}{2})} = C$$

$$C = -2$$

4. (4 points) Find the orthogonal trajectories of the family of curves $y = kx^3$, where k is an arbitrary constant. No need to solve explicitly for y.

Answer: $x^2 + 3y^2 = C$

$$\frac{y}{x^3} = k$$

$$\frac{x^3 \cdot \frac{dy}{dx} - y \cdot 3x^2}{x^6} = 0$$

$$x^3\frac{dy}{dx} - 3x^2 y = 0$$

$$\frac{dy}{dx} = \frac{3y}{x}$$

Orthogonal curves:

$$\frac{dy}{dx} = -\frac{x}{3y}$$

$$\int 3y\,dy = \int -x\,dx$$

$$\frac{3y^2}{2} = \frac{-x^2}{2} + C$$

$$x^2 + 3y^2 = C$$

Here are a couple of formulas I promised to give you:

$\int \sec x\,dx = \ln\lvert \sec x + \tan x \rvert + C$
$\int \csc x\,dx = -\ln\lvert \csc x + \cot x \rvert + C$