Test #1 Review Exercise Answers

1. $1-\frac{1}{e}$ (or $\frac{e-1}{e}$) (Set-up: $\int_1^2 e^{y-2}\,dy$)

2. $2\ln\left(\sqrt{2}-1\right)+3\sqrt{2}-\ln\left(2-\sqrt{3}\right)-\sqrt{3}-\ln\left(\sqrt{2}+1\right)$ (≈ 1.183);
   Set-up: $\int_{\pi/6}^{\pi/4}(\csc x-2\sin x)\,dx+\int_{\pi/4}^{3\pi/4}(2\sin x-\csc x)\,dx$

3. $\frac{16}{5\sqrt{3}}$ (Set-up: $\int_0^2 \sqrt{3}(2x-x^2)^2\,dx$)

4. $\frac{4\pi}{3}$ (Set-up: $\int_{-2}^2 \frac{1}{2}\pi\left(\frac{\sqrt{4-y^2}}{2}\right)^2 dy$)

5. $\frac{81\pi}{4}$
   Washer method set-up: $\pi\int_1^4\left(x^2-\left(\frac{1}{x}\right)^2\right)dx$
   Shell method set-up: $\int_{1/4}^1 2\pi y\left(4-\frac{1}{y}\right)dy+\int_1^4 2\pi y(4-y)\,dy$

6. 192π
   Shell method set-up: $\int_0^8 2\pi x\cdot x^{2/3}dx$
   Washer method set-up: $\int_0^4\left(\pi\cdot 8^2-\pi\left(y^{3/2}\right)^2\right)dy$

7. $\frac{13\pi}{2}$ (Shell method set-up: $\int_1^2 2\pi(3-x)x^2\,dx$)

8. $\frac{\pi}{6}$ (Shell method set-up: $\int_0^1 2\pi(1-y)(y-y^2)\,dy$)

9. 67.5 ft-lb ($k=15\frac{\text{lb}}{\text{ft}}$)

10. ($k=2\frac{\text{N}}{\text{m}}$)
    a. 9 J
    b. 20 m

11. 1470 J (Work to pull up chain: $\int_0^{10} 2\cdot 9.8\cdot y\,dy$ = 980 J ; Work to pull up 5-kg weight: $5\cdot 9.8\cdot 10$ = 490 J)

12. 400 ft-lb (Work to pull up rope: $\int_0^{20} 0.5\cdot y\,dy$ =100 ft-lb ; Work to pull up bucket: $15\cdot 20$ = 300 ft-lb)

13. 69688.9 J (Set-up: $\int_0^4 1000\cdot\left(\frac{y}{3}\right)^2\cdot 9.8\cdot(6-y)\,dy$)

14. 2369920 J (Set-up: $\int_{-2}^0 1000\cdot 10\cdot 2\sqrt{4-y^2}\cdot 9.8\cdot(3-y)\,dy$)

15. $\frac{8}{\sqrt{3}\pi}$

16. $\frac{e^2+1}{4(e-1)}$

17.

a. $x\tan^{-1}x-\frac{1}{2}\ln(x^2+1)+C$ (integration by parts)

b. $\frac{16}{3}\ln 4-\frac{28}{9}\approx 4.2825$ (integration by parts)

c. $x^2\sin x+2x\cos x-2\sin x+C$ (integration by parts)

d. $-\frac{\cos^5 x}{5}+\frac{2\cos^7 x}{7}-\frac{\cos^9 x}{9}+C$ (trig integral)

e. $\frac{\sec^5 x}{5}-\frac{2\sec^3 x}{3}+\sec x+C$ (trig integral)

f. $\frac{1}{8}x-\frac{1}{32}\sin 4x+C$ (trig integral)

g. $\ln\left|\frac{x}{4}+\frac{\sqrt{x^2-16}}{4}\right|+C$ (let $x=4\sec\theta$; the integral should simplify to $\int\sec\theta\,d\theta$)

h. $\frac{x}{\sqrt{9-x^2}}-\sin^{-1}\left(\frac{x}{3}\right)+C$ (let $x=3\sin\theta$; the integral should simplify to $\int\frac{\sin^2\theta}{\cos^2\theta}d\theta$)

i. $\ln\left|\frac{\sqrt{x^2+4x+8}}{2}+\frac{x+2}{2}\right|+C$ (complete square $(x+2)^2+4$; let $x+2=2\tan\theta$; the integral should simplify to $\int\sec\theta\,d\theta$)

j. $\ln(x^2+1)-\frac{1}{2(x^2+1)}-\frac{3}{2}\tan^{-1}x-\frac{3x}{2(x^2+1)}+C$ (the partial fraction decomposition looks like this: $\frac{2x}{x^2+1}+\frac{x-3}{(x^2+1)^2}$)

k. $-3\ln|x-1|+\ln|x+3|+\ln(x^2+4)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+C$ (the partial fraction decomposition looks like this: $-\frac{3}{x-1}+\frac{1}{x+3}+\frac{2x+1}{x^2+4}$)