Name: _____

Math 181 - Test #1 Info and Review Exercises

Fall 2018, Prof. Beydler

Test Info

- Date: Wednesday, September 26, 2018
- Will cover sections 6.1-6.5 and 7.1-7.4.
- You'll have the entire class to finish the test.
- This will be a 2-part test. Part 1 will be **no calculator**. Part 2 will be **scientific calculator only**.
- No notes, no books, no phones, no smart watches during the test.
- There will be a seating chart for the test.
- Where to get help as you're studying:
  - Office hours
  - TMARC, LAC, or other tutoring centers
  - E-mail me at dbeydler@mtsac.edu

Here are **some** of the formulas/concepts that you'll need to know:

Cross-sections

$$V = \int_a^b A(x)\,dx$$

Shell method

$$V = \int_a^b 2\pi(\text{radius})(\text{height})\,dx$$

Work

$$W = \int_a^b F(x)\,dx$$

Spring force

$$F = kx$$

Integration by parts

$$\int u\,dv = uv - \int v\,du$$

Trig formulas

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Trig substitutions

$\sqrt{a^2+x^2}$	$x = a\tan\theta$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
$\sqrt{a^2-x^2}$	$x = a\sin\theta$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$	$0 \le \theta < \frac{\pi}{2}$ (for $a > 0, x > 0$)

Partial fraction decomposition

Linear factor $(x-r)$: $\frac{A}{x-r}$

Repeated linear factor $(x-r)^m$: $\frac{A_1}{x-r} + \frac{A_2}{(x-r)^2} + \cdots + \frac{A_m}{(x-r)^m}$

Irreducible quadratic factor x^2+px+q: $\frac{Ax+B}{x^2+px+q}$

Repeated irreducible quadratic factor $(x^2+px+q)^n$: $\frac{B_1x+C_1}{(x^2+px+q)} + \frac{B_2x+C_2}{(x^2+px+q)^2} + \cdots + \frac{B_nx+C_n}{(x^2+px+q)^n}$

I'll give you these formulas if you need them:

$$\int \sec x\, dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\, dx = -\ln|\csc x + \cot x| + C$$

Review Exercises

Note: If you write up solutions to all of the review exercises listed below, and hand them in at the test, you can earn up to 2% extra credit towards your test! It is important to understand that these review exercises are not guaranteed to cover all of the potential problems on the test. Please review the notes and homework problems to fully prepare for the test.

Types of problems that will appear on Part 1 are labeled **NC** (for **N**o **C**alculator).

1. Find the area of the region(s) enclosed by the curves $y = 2 + \ln x$, $y = 1$, $y = 2$, and the y-axis.

2. Find the area of the region(s) enclosed by the curves $y = 2\sin x$, $y = \csc x$, $x = \frac{\pi}{6}$, and $x = \frac{3\pi}{4}$. (Recall that $\int \csc x\, dx = \ln|\csc x - \cot x| + C$. Also, this is a trig workout in disguise!)

3. The base of a solid is the region between $y = x^2 - 2x + 1$ and $y = -x^2 + 2x + 1$. Cross-sections perpendicular to the x-axis are equilateral triangles with one side in the base. Find the volume of the solid.

4. A solid has the following cross-sections perpendicular to the y-axis: half-disks (i.e. semicircles) whose diameters run from the y-axis to the curve $x = \sqrt{4 - y^2}$. Find the volume of the solid.

5. Find the volume of the solid generated by revolving the region bounded by the following curves about the x-axis.
   $y = x,\ \ y = \frac{1}{x},\ x = 4$

6. Find the volume of the solid generated by revolving the region bounded by the following curves about the y-axis.
   $y = x^{2/3},$ the x-axis, $x = 8$

7. Find the volume of the solid generated by revolving the region bounded by the following curves about the line $x = 3$.
   $y = x^2,\ \ y = 0,\ x = 1,\ x = 2$

8. Find the volume of the solid generated by revolving the region bounded by the following curves about the line $y = 1$.
   $x = y - y^2,\ \ x = 0$

9. A force of 30 lb is required to hold a spring stretched 2 ft beyond its natural length. How much work is done in stretching it from its natural length to 3 ft beyond its natural length?

10. Suppose it takes 4 J of work to stretch a spring from its natural length of 5 m to a length of 7 m.
    a. Find the work it takes to compress the spring to 2 m.

    b. How far beyond its natural length will a force of 40 N keep the spring stretched?

11. A 20-kg chain is 10-m long and hangs down from a 20-m-tall building. A 5-kg weight is attached to the end of the chain. Assuming the chain's mass is uniformly distributed, how much work is done lifting the chain to the top of the building?

12. You're thirsty, so you go to a 20-ft-deep well where there is a 15-lb bucket of water attached to the end of a rope. The rope is 0.5 lb/ft. How much work is needed to pull the bucket of water from the bottom of the well to the top of the well?

13. A tank in the shape of an inverted pyramid (tip down) with a square base holds water to a depth of 4 m. The height of the tank is 6 m, and the square base has dimensions 2 m by 2 m. How much work is required to pump the water to the top of the tank?

14. A 10-m-long trough with semicircular cross-sections is filled with water. The diameter of the semicircles is 4 m. How much work is required to pump the water to 3 meters above the top of the trough?

15. Find the average value of $f(x) = \csc^2 x$ on the interval $\left[\frac{\pi}{6}, \frac{2\pi}{3}\right]$.

16. Find the average value of $f(x) = x \ln x$ on the interval $[1, e]$.

17. Find the following integrals. (**NC**)
    a. $\int \tan^{-1} x \, dx$

b. $\int_1^4 \sqrt{x} \ln x \, dx$

c. $\int x^2 \cos x \, dx$

d. $\int \sin^5 x \cos^4 x \, dx$

e. $\int \tan^5 x \sec x \, dx$

f. $\int \sin^2 x \cos^2 x \, dx$

g. $\int \frac{dx}{\sqrt{x^2-16}}$

h. $\int \frac{x^2}{(9-x^2)^{3/2}}\,dx$

i. $\int \frac{dx}{\sqrt{x^2+4x+8}}$

j. $\int \frac{2x^3+3x-3}{x^4+2x^2+1} dx$

k. $\int \frac{-5x^2-12x-43}{(x-1)(x+3)(x^2+4)}\,dx$