Test #1 (Part 2, Calculator Okay) Name: ____________________

Math 181, Prof. Beydler Wednesday, September 26, 2018

Directions: Show all work. No books or notes. A **scientific calculator** is allowed. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. Good luck!

1. (6 points) Find the area of the region(s) enclosed by $y = \sqrt{x}$, $y = x$, and $0 \le x \le 2$.

Answer: $\frac{7-4\sqrt{2}}{3}$

Int pts:

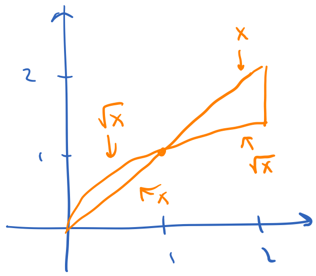
$\sqrt{x} = x$

$x = x^2$

$0 = x^2 - x$

$0 = x(x-1)$

$x=0 \quad x=1$



$$A = \int_0^1 (\sqrt{x} - x)\,dx + \int_1^2 (x - \sqrt{x})\,dx$$

$$= \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - \frac{2}{3}x^{3/2}\right]_1^2$$

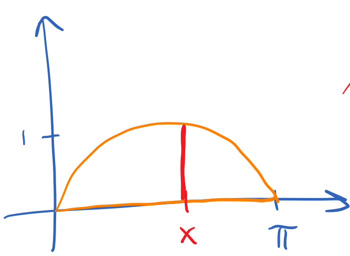
$$= \frac{2}{3} - \frac{1}{2} + \left(2 - \frac{2}{3}(2)^{3/2} - \left(\frac{1}{2} - \frac{2}{3}\right)\right)$$

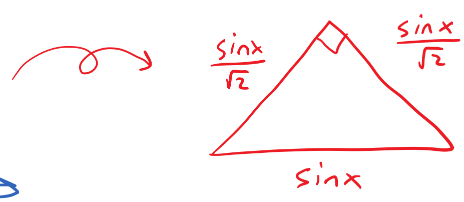
$$= \frac{1}{6} + 2 - \frac{4\sqrt{2}}{3} + \frac{1}{6}$$

$$= \frac{7}{3} - \frac{4\sqrt{2}}{3}$$

2. (5 points) The base of a solid is the region between $y = \sin x$ and the x-axis where $0 \le x \le \pi$. Cross-sections perpendicular to the x-axis are isosceles triangles with hypotenuse along the base. Find the volume of the solid.

Answer: $\frac{\pi}{8}$





$$A(x) = \frac{1}{2}\left(\frac{\sin x}{\sqrt{2}}\right)^2 = \frac{1}{4}\sin^2 x$$

$$V = \int_0^\pi \frac{1}{4}\sin^2 x\,dx = \int_0^\pi \frac{1}{8}(1 - \cos 2x)\,dx$$

$$= \frac{1}{8}\left(x - \frac{1}{2}\sin 2x\right)\Big]_0^\pi$$

$$= \frac{1}{8}(\pi)$$

3. (4 points) **Set up (but do not evaluate) an integral** to find the volume of the solid generated by revolving the region bounded by the following curves about the line $x = 2$.

$x = y^2,\ x = 2 - y^2$

Int pts:

$y^2 = 2 - y^2$

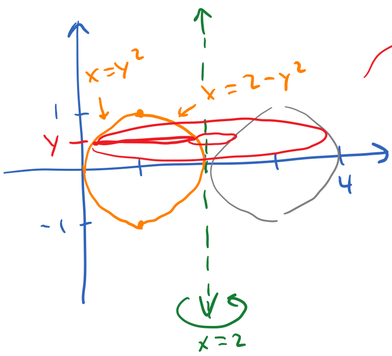
$2y^2 = 2$

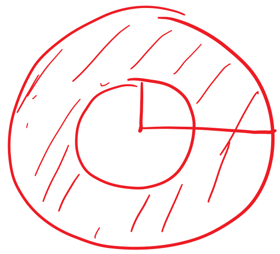
$y^2 = 1$

$y = \pm 1$

Answer: $\int_{-1}^{1} \left(\pi(2-y^2)^2 - \pi y^4\right) dy$

$\left(\text{OR } \pi\int_{-1}^{1}(4-4y^2)\,dy\right)$





$A(y) = \pi(2-y^2)^2 - \pi\underbrace{(2-(2-y^2))^2}_{y^4}$

4. (4 points) **Set up (but do not evaluate) an integral** to find the volume of the solid generated by revolving the region bounded by the following curves about the y-axis.

$y = 4x - x^2,\ \ y = 3$

Int pts:

$4x - x^2 = 3$

$0 = x^2 - 4x + 3$

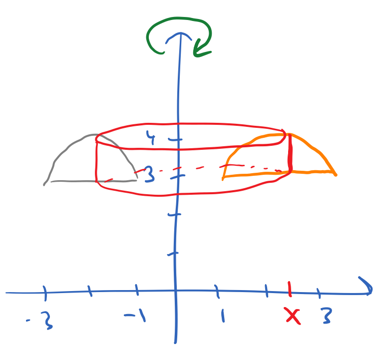
$0 = (x-1)(x-3)$

$x = 1 \qquad x = 3$

Answer: $\int_{1}^{3} 2\pi x\,(4x - x^2 - 3)\,dx$

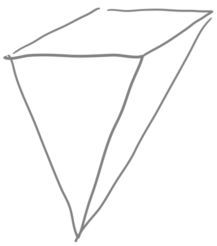
radius $= x$

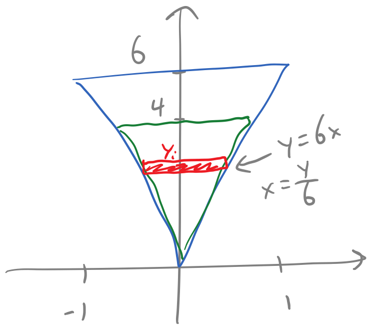
height $= 4x - x^2 - 3$

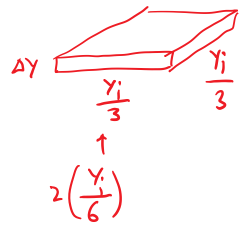


5. (6 points) A tank in the shape of an inverted pyramid (tip down) with a square base holds water to a depth of 4 m. The height of the tank is 6 m, and the square base has dimensions 2 m by 2 m. How much work is required to pump the water to the top of the tank? Use 1000 kg/m^3 for the mass density of water, and be sure to write the units for your answer.

Answer: 69689 J







$$V_i = \left(\frac{y_i}{3}\right)^2 \Delta y$$

$$F_i = 1000 \left(\frac{y_i}{3}\right)^2 \Delta y \cdot 9.8$$

$$W_i = \frac{9800}{9} y_i^2 \Delta y \,(6 - y_i)$$

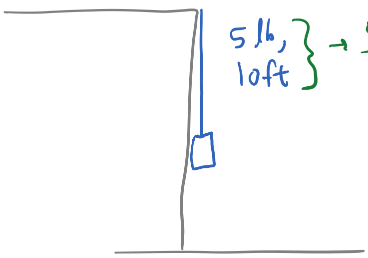
$$W = \int_0^4 \frac{9800}{9} \underbrace{y^2(6-y)}_{6y^2-y^3} \, dy$$

$$= \frac{9800}{9}\left[2y^3 - \frac{y^4}{4}\right]_0^4$$

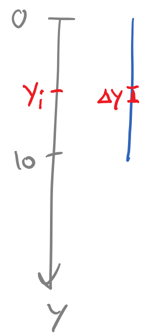
$$= \frac{9800}{9}(128 - 64)$$

6. (5 points) A 5-lb rope is 10-ft long and hangs down from a 20-ft-tall building. A 6-lb weight is attached to the end of the rope. Assuming the rope's mass is uniformly distributed, how much work is done lifting the rope and the weight to the top of the building? Be sure to write the units for your answer.

Answer: 85 ft-lb



$\left.\begin{matrix} 5\text{ lb}, \\ 10\text{ ft} \end{matrix}\right\} \rightarrow \frac{5\text{ lb}}{10\text{ ft}} = 0.5\ \text{lb/ft}$



$F_i = (0.5\ \text{lb/ft})\,\Delta y$

$W_i = 0.5\,\Delta y \cdot y_i$

$W_{rope} = \int_0^{10} 0.5\,y\,dy$

$= 0.25\,y^2\Big]_0^{10}$

$= 25$ ft-lb

$W_{weight} = (6\text{ lb})(10\text{ ft})$

$= 60$ ft-lb

$60 + 25 = 85$ ft-lb

7. (4 points) Find the average value of $f(x) = e^{\cos x}\sin x$ on the interval $\left[0, \frac{\pi}{2}\right]$.

Answer: $\frac{2}{\pi}(e-1)$

$f_{ave} = \frac{1}{\frac{\pi}{2} - 0} \int_0^{\frac{\pi}{2}} e^{\cos x} \sin x\, dx$

$u = \cos x$
$du = -\sin x\, dx$
$-du = \sin x\, dx$

$x = 0: u = \cos 0 = 1$
$x = \frac{\pi}{2}: u = \cos\frac{\pi}{2} = 0$

$= \frac{2}{\pi} \int_1^0 -e^u\, du$

$= \frac{2}{\pi} \int_0^1 e^u\, du$

$= \frac{2}{\pi} \left[e^u\right]_0^1$

8. (0 points) How many hours of sleep did you get last night? $-\infty$

Note: Be sure to double-check your work. And don't forget to turn in your homework! ☺