_____ / 54 total points (both parts)

**Test #1 (Part 1, No Calculator)**

Name: _____

Math 181, Prof. Beydler

Wednesday, September 26, 2018

**Directions:** Show all work. No calculator, books, or notes. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. When you're finished with Part 1, please turn it in, take a bathroom break, get your calculator out, and start Part 2. Good luck!

1. (4 points) Find the following integral.

$$\int \sin^{-1} x \, dx$$

$= x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}dx$

$\sin^{-1}x$ (+) → $1$; $\frac{1}{\sqrt{1-x^2}}$ (−) → $x$

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$= x\sin^{-1}x + \frac{1}{2}\int \frac{1}{\sqrt{u}}du$

$= x\sin^{-1}x + \frac{1}{2}(2\sqrt{u}) + C$

Answer: $x\sin^{-1}x + \sqrt{1-x^2} + C$

2. (4 points) Find the following integral.

$$\int \sin^3 x \cos^4 x \, dx$$

$= \int \sin^2 x \cos^4 x \cdot \sin x \, dx$

$= \int (1-\cos^2 x)\cos^4 x \cdot \sin x \, dx$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$= \int (1-u^2)u^4 \cdot (-du)$

$= \int (u^6 - u^4)du$

$= \frac{u^7}{7} - \frac{u^5}{5} + C$

Answer: $\frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$

3. (6 points) Find the following integral.

$$\int \frac{dx}{\sqrt{x^2+4x+8}}$$

Answer: $\ln\left|\frac{\sqrt{x^2+4x+8}}{2}+\frac{x+2}{2}\right|+C$

$= \int \frac{dx}{\sqrt{(x+2)^2+4}}$

$x^2+4x+4+8-4$

$x+2 = 2\tan\theta \quad \leftarrow -\frac{\pi}{2} < x < \frac{\pi}{2}$

$dx = 2\sec^2\theta\, d\theta$

$= \int \frac{2\sec^2\theta\, d\theta}{\sqrt{4\tan^2\theta+4}}$

$\sqrt{4(\tan^2\theta+1)}$

$= \sqrt{4\sec^2\theta}$

$= 2\sec\theta$

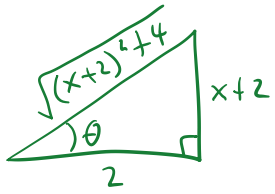
$= \int \frac{2\sec^2\theta}{2\sec\theta}\, d\theta$

$= \int \sec\theta\, d\theta$

$= \ln|\sec\theta+\tan\theta|+C$

$= \ln\left|\frac{\sqrt{(x+2)^2+4}}{2}+\frac{x+2}{2}\right|+C$

$\tan\theta = \frac{x+2}{2}$



4. (6 points) Find the following integral.

$$\int \frac{x^3 + 4x^2 + 2x + 3}{x^4 + x^2}\,dx$$

Answer: $2\ln|x| - \frac{3}{x} - \frac{1}{2}\ln|x^2+1| + \tan^{-1}x + C$

$$= \int \left(\frac{2}{x} + \frac{3}{x^2} + \frac{-x+1}{x^2+1}\right) dx$$

$$= 2\ln|x| - \frac{3}{x} - \int \frac{x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx$$

$u = x^2+1$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$
$\frac{1}{2}\int \frac{1}{u}\,du$

$$= 2\ln|x| - \frac{3}{x} - \frac{1}{2}\ln|x^2+1| + \tan^{-1}x + C$$

$$\frac{x^3+4x^2+2x+3}{x^2(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$$

$$x^3+4x^2+2x+3 = Ax(x^2+1) + B(x^2+1) + (Cx+D)x^2$$

$$= Ax^3 + Ax + Bx^2 + B + Cx^3 + Dx^2$$

$$= (A+C)x^3 + (B+D)x^2 + Ax + B$$

$$\begin{cases} A+C=1 \\ B+D=4 \\ A=2 \\ B=3 \end{cases} \rightarrow \begin{array}{l} A=2 \\ B=3 \\ C=-1 \\ D=1 \end{array}$$

Here are a couple of formulas I promised to give you:

$\int \sec x\,dx = \ln|\sec x + \tan x| + C$
$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$