

## Final Exam Review Exercise Answers – Math 181

1. 2 (set up:  $\int_{\pi/4}^{3\pi/4} (\sin^2 x - \cos^2 x) dx + \int_{3\pi/4}^{5\pi/4} (\cos^2 x - \sin^2 x) dx$ )
2.  $\frac{8}{15}$  (set up:  $\int_0^2 \frac{(2x-x^2)^2}{2} dx$ )
3. Using washers:  $\int_0^1 \left[ \pi(2 - (-1))^2 - \pi(2\sqrt{x} - (-1))^2 \right] dx = \frac{10\pi}{3}$   
Using shells:  $\int_0^2 2\pi(y - (-1)) \left( \frac{y^2}{4} - 0 \right) dy = \frac{10\pi}{3}$
4. Using shells:  $\int_0^1 2\pi(1 - y)((y - y^3) - 0) dy = \frac{7\pi}{30}$
5. 80 ft-lb (set up:  $\int_0^4 10x dx$ )
6. 514.5 J (set up:  $\int_0^5 3 \cdot 9.8 \cdot y dy + 3 \cdot 9.8 \cdot 5$ )
7. 58643.1 ft-lb (set up:  $\int_0^8 \pi \left( \frac{y}{2} \right)^2 \cdot 62.5 \cdot (13 - y) dy$ )
8.  $2 - \frac{4}{\pi}$  (set up:  $\frac{1}{2} \int_0^{\pi/2} x^2 \sin x dx$ , use integration by parts)
9.
  - a. 1 (Use integration by parts with  $u = \cos^{-1} x$  and  $dv = dx$ )
  - b.  $\frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + C$  (Save one  $\sec^2 x$  with  $dx$ , convert other  $\sec^2 x$  to  $1 + \tan^2 x$ , let  $u = \tan x$ )
  - c.  $\frac{x}{4\sqrt{4-x^2}} + C$  (Trig substitution with  $x = 2 \sin \theta$ )
  - d.  $\frac{(4+x^2)^{3/2}}{3} - 4\sqrt{4+x^2} + C$
  - e.  $9x + 2 \ln|x| + \frac{1}{x} + 7 \ln|x-1| + C$
  - f.  $\ln(x^2 + 1) + \tan^{-1} x - 2 \ln|x-1| - \frac{1}{x-1} + C$  (Note:  $\frac{-2x+4}{(x^2+1)(x-1)^2} = \frac{2x+1}{x^2+1} - \frac{2}{x-1} + \frac{1}{(x-1)^2}$ )
  - g. Diverges (first step looks like this:  $\lim_{t \rightarrow 0^-} \int_{-1}^t \frac{1}{x^2} dx + \lim_{t \rightarrow 0^+} \int_t^2 \frac{1}{x^2} dx$ )
  - h. Converges to 0 (first step looks like this:  $\lim_{t \rightarrow -\infty} \int_t^0 2xe^{-x^2} dx + \lim_{t \rightarrow \infty} \int_0^t 2xe^{-x^2} dx$ )
  - i.  $3\sqrt[3]{x^2}e^{\sqrt[3]{x}} - 6\sqrt[3]{x}e^{\sqrt[3]{x}} + 6e^{\sqrt[3]{x}} + C$  (substitution with  $u = \sqrt[3]{x}$ ,  $3u^2 du = dx$ )
  - j.  $\frac{1}{2} (\ln|\sqrt{x^2+1}-1| - \ln|\sqrt{x^2+1}+1|) + C$  (substitution with  $u = \sqrt{x^2+1}$ )  
or  $\ln \left| \frac{x}{\sqrt{x^2+1}+1} \right| + C$  (trig substitution with  $x = \tan \theta$ )
  - k.  $x - \ln|e^x + 1| + C$  (substitution with  $u = e^x + 1$ )
10.  $\frac{x+3}{\sqrt{4x^4-3x-1}} > \frac{x}{\sqrt{4x^4}} = \frac{x}{2x^2} = \frac{1}{2x}$  so since  $\int_2^\infty \frac{1}{2x} dx$  diverges,  $\int_2^\infty \frac{x+3}{\sqrt{4x^4-3x-1}} dx$  diverges by comparison.
11.  $\frac{595}{144}$  (set up:  $\int_2^3 \sqrt{1 + \left( \frac{x^3}{4} - \frac{1}{x^3} \right)^2} dx$ )
12.  $8\pi$  (set up:  $\int_{-1}^1 2\pi\sqrt{4-x^2} \sqrt{1 + \left( -\frac{x}{\sqrt{4-x^2}} \right)^2} dx$ )

13. 246960 N (set up with a coordinate system that has  $x$ -axis along bottom of trapezoid and  $y$ -axis cutting trapezoid in half:  $P_i = \rho g d = 1000 \cdot 9.8 \cdot (3 - y_i)$  and  $A_i = 2 \cdot \left(\frac{15-y_i}{5}\right) \Delta y$ ;

$$\sum_{i=1}^n 1000 \cdot 9.8 \cdot (3 - y_i) \cdot 2 \cdot \left(\frac{15-y_i}{5}\right) \Delta y; \text{ integral set up: } \int_0^3 1000 \cdot 9.8 \cdot (3 - y) \cdot 2 \cdot \left(\frac{15-y}{5}\right) dy$$

14.  $\left(1, \frac{3}{5}\right)$  (set up:  $A = \int_0^2 (x - (x^2 - x)) dx = \frac{4}{3}$ ,  $\bar{x} = \frac{1}{A} \int_0^2 x(x - (x^2 - x)) dx$ ,  $\bar{y} = \frac{1}{A} \int_0^2 \frac{x^2}{2} (x - (x^2 - x)) dx$ )

15.  $y'' + 3y' + y = (-t \sin t + 2 \cos t) + 3(t \cos t + \sin t) + t \sin t = 3t \cos t + 3 \sin t + 2 \cos t$ ;  
 $y(0) = 0 \cdot \sin 0 = 0$ ;  $y'(0) = 0 \cdot \cos 0 + \sin 0 = 0$

16.

- a.  $1 < y < 2$
- b.  $y < 1$  or  $y > 2$
- c.  $y = 1, y = 2$

17.

- a.  $y = e^{-xe^{-x}-e^{-x}+1}$  (set up: separate variables  $\int \frac{1}{y} dy = \int x e^{-x} dx$ )
- b.  $y = \frac{1}{2} e^{2-\cos x}$  (set up: separate variables  $\int \frac{1}{y} dy = \int \sin x dx$ )

18.  $\ln|y| - \frac{y^2}{2} = \frac{x^2}{2} + C$

19. 9.93% alcohol (set up:  $\frac{dA}{dt} = 2 - \left(\frac{A}{100}\right) \cdot 20$ ;  $A(t) = 10 \pm K e^{-t/5}$ ;  $K = -4$ ;  $A(t) = 10 - 4e^{-t/5}$ )

20.  $y - 1 = \frac{4}{\pi} \left(x - \frac{\pi^2}{16}\right)$  (or  $y = \frac{4}{\pi} x - \frac{\pi}{4} + 1$ )

21.  $\sqrt{1+e^2} + \frac{1}{2} \ln\left(\frac{\sqrt{1+e^2}-1}{\sqrt{1+e^2}+1}\right) - \sqrt{2} - \frac{1}{2} \ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)$  (set up:  $\int_0^1 \sqrt{1+e^{2t}} dt$ , substitution with  $u = \sqrt{1+e^{2t}}$ ;  
 you'll get  $\int_{\sqrt{2}}^{\sqrt{1+e^2}} \frac{u^2}{u^2-1} du$ ; divide, then do a partial fraction decomposition)

22.

- a.  $4\pi$  (set up:  $2 \int_0^{\pi/2} \frac{1}{2} (4 \cos \theta)^2 d\theta$ ; or it's just the area of a circle with radius 2!)
- b.  $\frac{\pi}{4}$  (set up:  $2 \left( \int_0^{\pi/6} \frac{1}{2} (1 + \sin \theta)^2 d\theta - \int_0^{\pi/6} \frac{1}{2} (3 \sin \theta)^2 d\theta + \int_{3\pi/2}^{2\pi} \frac{1}{2} (1 + \sin \theta)^2 d\theta \right)$ )
- c.  $\frac{\pi}{12}$  (set up:  $2 \int_0^{\pi/6} \frac{1}{2} (\cos 3\theta)^2 d\theta$ )

23.  $2$  (set up:  $\int_0^{\pi} \sqrt{\left(\sin^2 \frac{\theta}{2}\right)^2 + \left(\sin \frac{\theta}{2} \cos \frac{\theta}{2}\right)^2} d\theta$ )

24.  $\frac{3}{5\sqrt{3}}$

25. Horizontal:  $\theta = \frac{3\pi}{4}$  and  $\theta = \frac{7\pi}{4}$ ; Vertical:  $\theta = \frac{\pi}{4}$  and  $\theta = \frac{5\pi}{4}$  ( $\frac{dy}{dx} = \frac{e^\theta \sin \theta + e^\theta \cos \theta}{e^\theta \cos \theta - e^\theta \sin \theta}$ )

26.

- a. converges to  $e^\pi$

- b. diverges
- c. converges to 0
- d. diverges

27.  $\frac{3}{2}$

28.

- a. converges to  $\frac{1}{e^4 - e^2}$  (geometric series with  $a = \frac{1}{e^4}$  and  $r = \frac{1}{e^2}$ )
- b. converges (by Alternating Series Test)
- c. diverges (by Integral Test)
- d. diverges (by Test for Divergence)
- e. converges to  $\frac{5}{6}$  (telescoping sums, note that  $\frac{2}{(n+1)(n+3)} = \frac{1}{n+1} - \frac{1}{n+3}$ )
- f. converges (by Root Test or Ratio Test)
- g. diverges (by Limit Comparison Test with  $\sum \frac{1}{2\sqrt{n}}$ )
- h. converges (by Comparison Test with  $\sum \frac{2}{n^{3/2}}$ )

29.

- a. converges (by Ratio Test where  $L = \frac{2}{3} < 1$ )
- b. converges (by Root Test  $L = 0 < 1$ )
- c. converges conditionally ( $\sum \left| (-1)^n \frac{3}{n+1} \right|$  diverges by Limit Comparison Test with  $\sum \frac{1}{n}$  and  $\sum (-1)^n \frac{3}{n+1}$  converges by Alternating Series Test)
- d. converges absolutely ( $\sum_{n=1}^{\infty} \left| \frac{(-2)^{n+1}}{n+5^n} \right| = \sum_{n=1}^{\infty} \frac{2^{n+1}}{n+5^n}$  converges by Comparison Test with  $\sum 2 \cdot \left(\frac{2}{5}\right)^n$ )

30.

- a. Interval:  $(-2, 0]$ , Radius: 1 (Note: diverges at  $x = -2$  by Limit Comparison test with  $\sum \frac{1}{n}$ , converges at  $x = 0$  by Alternating Series Test)
- b. Interval:  $(-\infty, \infty)$ , Radius:  $\infty$

31.  $\sum_{n=0}^{\infty} \left(-\frac{1}{2^{n+1}} x^{n+2}\right)$  and interval of convergence  $(-2, 2)$  (or reindex to get  $\sum_{n=2}^{\infty} \left(-\frac{1}{2^{n-1}} x^n\right)$ )

32.  $\ln 4 - \sum_{n=1}^{\infty} \frac{1}{n \cdot 4^n} x^n$  and radius of convergence is 4 (Hint:  $\ln(4-x) = \ln 4 \left(1 + \left(-\frac{x}{4}\right)\right) = \ln 4 + \ln\left(1 + \left(-\frac{x}{4}\right)\right)$ )

33.  $\sum_{n=0}^{\infty} \frac{2(\ln 2)^n}{n!} (x-1)^n$  and radius of convergence is  $\infty$

34.  $1 + \frac{1}{2}(x-1) + \sum_{n=2}^{\infty} \frac{(-1)^{n-1} \cdot 1 \cdot 3 \cdot 5 \cdots (2n-3)}{2^n n!} (x-1)^n$  and radius of convergence is 1

35.  $\sum_{n=0}^{\infty} \frac{2^n}{n!} x^{2n+1}$  and radius of convergence is  $\infty$

36.  $\frac{1}{2} + \sum_{n=1}^{\infty} \frac{1 \cdot 4 \cdot 7 \cdots (3n-2)}{2 \cdot n! \cdot 24^n} x^n$  and radius of convergence is 8

37.  $C + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!(4n+3)} x^{4n+3}$

38. 0.003 (Note: the antiderivative is  $\frac{x}{3} - \frac{x^5}{5} + \frac{x^7}{14} - \frac{x^9}{54} + \dots$ )

39.  $-1$  (Note: at some point you'll get  $\lim_{x \rightarrow \infty} \left(-1 + \frac{1}{2x^2} - \frac{1}{6x^4} + \frac{1}{24x^6} - \dots\right)$ )

40.  $x^2 - \frac{2}{3}x^4 + \frac{23}{45}x^6 + \dots$  (Note:  $(\tan^{-1} x)^2 = (\tan^{-1} x)(\tan^{-1} x)$ )

41.  $x + \frac{1}{2}x^2 + \frac{5}{6}x^3 + \dots$  (Note:  $\frac{\ln(1+x)}{1-x} = \ln(1+x) \cdot \frac{1}{1-x}$ ; Or, you can do long division...)

42.  $T_3(x) = x - \frac{x^3}{2!}$

43.  $T_3(x) = (x - 1) + \frac{(x-1)^2}{2} - \frac{(x-1)^3}{6}$ ;  $T_3(x)$  is accurate within 0.041667 as long as  $0.5 \leq x \leq 1.5$  (Note:  
 $f^{(4)}(x) = \frac{2}{x^3}$ ; use  $M = 16$ )