9.1/9.3 – Practice

1. Find the general solution to the following differential equation.

$$y' = y^2 + 2xy^2$$

$$\frac{dy}{dx} = y^2(1+2x)$$

$$\int \frac{1}{y^2}\,dy = \int (1+2x)\,dx$$

$$-\frac{1}{y} = x + x^2 + C$$

$$\boxed{y = \frac{-1}{x^2+x+C}}$$

2. Using your answer from the previous problem, solve the initial value problem:

$$y' = y^2 + 2xy^2, \quad y(0) = 2$$

$$\frac{dy}{dx} = y^2(1+2x)$$

$$\int \frac{1}{y^2}\,dy = \int (1+2x)\,dx$$

$$-\frac{1}{y} = x + x^2 + C$$

Find C:

$$-\frac{1}{2} = 0 + 0^2 + C$$

$$C = -\frac{1}{2}$$

$$-\frac{1}{y} = x + x^2 - \frac{1}{2}$$

$$\frac{1}{y} = -x - x^2 + \frac{1}{2}$$

$$y = \frac{1}{-x-x^2+\frac{1}{2}}$$

$$\boxed{y = \frac{2}{-2x-2x^2+1}}$$

3. Find the orthogonal trajectories of the family of curves below.

$$y = cx^2$$

① Find slopes of curves $\left(\frac{dy}{dx}\right)$

$$\frac{y}{x^2} = c$$

$$\frac{d}{dx}\left(\frac{y}{x^2}\right) = \frac{d}{dx}(c)$$

$$\frac{x^2\frac{dy}{dx} - y\cdot 2x}{(x^2)^2} = 0$$

$$x^2\frac{dy}{dx} - 2xy = 0$$

$$\frac{dy}{dx} = \frac{2y}{x}$$

Slopes of $y = cx^2$

② Slopes of orthogonal curves will be negative reciprocals

$$\frac{dy}{dx} = \frac{-x}{2y}$$

③ Solve DE

$$\int 2y\,dy = \int (-x)\,dx$$

$$y^2 = -\frac{x^2}{2} + C$$

$b = 2C$

$$\boxed{x^2 + 2y^2 = b}$$

ellipses

Q: What has four wheels and flies?