1. Use the Trapezoidal Rule with $n = 4$ steps to approximate $\int_{-1}^{1}(x^2 - 1)dx$ and estimate the error in the approximation (i.e. find an upper bound for $|E_T|$). Evaluate the integral directly and find $|E_T|$.

$$\Delta x = \frac{1-(-1)}{4} = \frac{1}{2}$$

x_i	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
$f(x_i)$	0	$\frac{-3}{4}$	-1	$\frac{-3}{4}$	0

$$\int_{-1}^{1}(x^2-1)dx \approx \frac{(1/2)}{2}\left[0+2\left(\frac{-3}{4}\right)+2(-1)+2\left(\frac{-3}{4}\right)+0\right]$$

$$= \boxed{-1.25}$$

Approximate area using Trapezoidal Rule

Find upper bound for $|E_T|$:

$$|E_T| \le \frac{M(b-a)^3}{12n^2}$$

$$= \frac{(2)(1-(-1))^3}{12(4)^2}$$

$$= \boxed{\frac{1}{12}} \leftarrow \text{Upper bound for } |E_T|$$

$$(= 0.08\overline{3})$$

$f(x) = x^2 - 1$

$f'(x) = 2x$

$f''(x) = 2$

On $[-1, 1]$, $|f''| \le 2$

Evaluate directly:

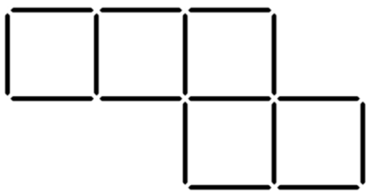
$$\int_{-1}^{1}(x^2-1)dx = \left[\frac{x^3}{3} - x\right]_{-1}^{1} = \left(\frac{1}{3}-1\right)-\left(-\frac{1}{3}-(-1)\right) = \boxed{-\frac{4}{3}}$$

Find $|E_T|$:

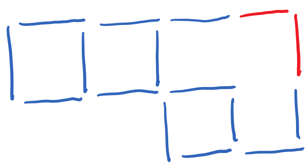
$$|E_T| = |\text{True area} - \text{Estimated area}| = \left|-\frac{4}{3} - (-1.25)\right| = \boxed{0.08\overline{3}}$$

Q: What gets wet when drying?

Bonus puzzle!



Move 2 line segments to make 4 squares.
One square will be larger than the other 3.



OR