

1. Evaluate each integral.

a) $\int \frac{\cos x}{\sin^2 x - \sin x} dx$

$$= \int \frac{1}{u^2 - u} du$$

$$= \int \left(\frac{-1}{u} + \frac{1}{u-1} \right) du$$

$$= -\ln|u| + \ln|u-1| + C$$

$$= \boxed{-\ln|\sin x| + \ln|\sin x - 1| + C}$$

$$(OR \ln \left| \frac{\sin x - 1}{\sin x} \right| + C, \text{ OR } \ln|1 - \csc x| + C)$$

$$u = \sin x \\ du = \cos x dx$$

$$\frac{1}{u^2 - u} = \frac{1}{u(u-1)} = \frac{A}{u} + \frac{B}{u-1}$$

$$1 = A(u-1) + Bu$$

$$u=0: 1 = -A \rightarrow A = -1$$

$$u=1: 1 = B$$

b) $\int \frac{\tan^3 x}{\cos^3 x} dx$

$$= \int \frac{\sin^3 x}{\cos^6 x} dx$$

$$= \int \frac{\sin^2 x}{\cos^6 x} \sin x dx$$

$$= \int \frac{1 - \cos^2 x}{\cos^6 x} \sin x dx$$

$$= \int \frac{1 - u^2}{u^6} \cdot (-du)$$

$$= -\int (u^{-6} - u^{-4}) du$$

$$= -\left(\frac{u^{-5}}{-5} - \frac{u^{-3}}{-3} \right) + C$$

$$\frac{\tan^3 x}{\cos^3 x} = \frac{\left(\frac{\sin^3 x}{\cos^3 x} \right)}{\cos^3 x} = \frac{\sin^3 x}{\cos^6 x}$$

$$u = \cos x \\ du = -\sin x dx \\ -du = \sin x dx$$

$$= \boxed{\frac{1}{5 \cos^5 x} - \frac{1}{3 \cos^3 x} + C}$$

$$\begin{aligned}
 & \text{b) } \int (\sqrt{x} + 2)^{10} dx \\
 & = \int u^{10} \cdot 2(u-2) du \\
 & = 2 \int (u^{11} - 2u^{10}) du \\
 & = 2 \left(\frac{u^{12}}{12} - \frac{2u^{11}}{11} \right) + C \\
 & = \boxed{\frac{(\sqrt{x} + 2)^{12}}{6} - \frac{4(\sqrt{x} + 2)^{11}}{11} + C}
 \end{aligned}$$

$u = \sqrt{x} + 2 \rightarrow u - 2 = \sqrt{x}$
 $du = \frac{1}{2\sqrt{x}} dx$
 $du = \frac{1}{2(u-2)} dx$
 $2(u-2) du = dx$

Q: In a zoo with only birds and mammals, there are 30 heads and 80 legs. How many birds and how many mammals are at this zoo?