Trigonometric Substitutions

To evaluate integrals involving $\sqrt{a^2+x^2}$, $\sqrt{a^2-x^2}$, and $\sqrt{x^2-a^2}$, it sometimes helps to make what's called a **trigonometric substitution**.

Ex 1.

$$\int \frac{x^2\,dx}{\sqrt{9-x^2}}$$

How do you know what trig substitution to make? Here's a table you'll want to know:

$\sqrt{a^2+x^2}$	$x = a\tan\theta$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
$\sqrt{a^2-x^2}$	$x = a\sin\theta$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$	$0 \le \theta < \frac{\pi}{2}$ or $\pi \le \theta < \frac{3\pi}{2}$

Ex 2.

$$\int \frac{dx}{\sqrt{4+x^2}}$$

Ex 3.

$$\int \frac{dx}{\sqrt{25x^2-4}}$$

Here's how you would start the following integral: $\int \frac{x}{\sqrt{3-2x-x^2}}\,dx$

Practice

1. Evaluate each integral.

a) $\int \frac{\sqrt{9-x^2}}{x^2}\,dx$

b) $\int \frac{dx}{\sqrt{x^2-a^2}}$ (Assume a is a positive constant.)

c) $\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$

Q: How can half of 12 be 7?