

1. Evaluate each integral.

a) $\int \cos^5 x \, dx$

$$\begin{aligned} &= \int (\cos^2 x)^2 \cdot \cos x \, dx \\ &= \int (1 - \sin^2 x)^2 \cdot \cos x \, dx \\ &= \int (1 - u^2)^2 \cdot du \end{aligned}$$

$\cos^2 x = 1 - \sin^2 x$
 $u = \sin x$
 $du = \cos x \, dx$

$$\begin{aligned} &= \int (1 - 2u^2 + u^4) \, du \\ &= u - \frac{2}{3}u^3 + \frac{1}{5}u^5 + C \\ &= \boxed{\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C} \end{aligned}$$

b) $\int \sin^2 x \, dx$

$$\begin{aligned} &= \int \frac{1 - \cos 2x}{2} \, dx \\ &= \frac{1}{2} \int (1 - \cos 2x) \, dx \\ &= \frac{1}{2} \left(x - \frac{1}{2} \sin 2x \right) + C \\ &= \boxed{\frac{1}{2}x - \frac{1}{4}\sin 2x + C} \end{aligned}$$

$$c) \int \tan^4 x \, dx$$

$$= \int (\tan^2 x)^2 \, dx$$

$$= \int (\sec^2 x - 1)^2 \, dx$$

$$= \int (\sec^4 x - 2\sec^2 x + 1) \, dx \quad \rightarrow$$

$$= \frac{\tan^3 x}{3} + \tan x - 2\tan x + x + C$$

$$= \boxed{\frac{\tan^3 x}{3} - \tan x + x + C}$$

$$\begin{aligned} \int \sec^4 x \, dx &= \int \sec^2 x \cdot \sec^2 x \, dx \\ &= \int (\tan^2 x + 1) \cdot \sec^2 x \, dx \quad \left. \begin{array}{l} u = \tan x \\ du = \sec^2 x \, dx \end{array} \right\} \\ &= \int (u^2 + 1) \, du \\ &= \frac{u^3}{3} + u \\ &= \frac{\tan^3 x}{3} + \tan x \end{aligned}$$

Q: The more you take, the more you leave behind. What are they?