

## Average Value of a Function

Q: How do we find the **average value** of a continuous function?

For example, let's find the average value of  $f(x)$  over the interval  $[a, b]$ . Note that  $\Delta x = \frac{b-a}{n}$  so that  $n = \frac{b-a}{\Delta x}$ . Well then,

Average value of  $f$  over  $[a, b]$

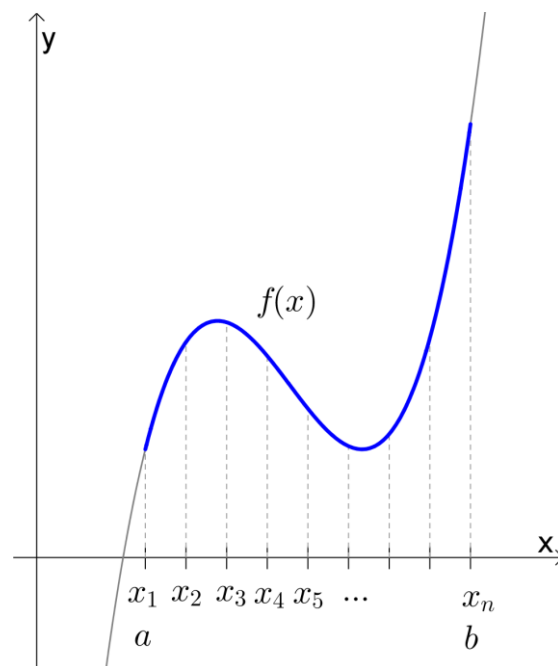
$$= \lim_{n \rightarrow \infty} \frac{f(x_1) + f(x_2) + \cdots + f(x_n)}{n}$$

$$= \lim_{n \rightarrow \infty} \frac{f(x_1) + f(x_2) + \cdots + f(x_n)}{\left(\frac{b-a}{\Delta x}\right)}$$

$$= \lim_{n \rightarrow \infty} \frac{(f(x_1) + f(x_2) + \cdots + f(x_n))\Delta x}{b-a}$$

$$= \frac{1}{b-a} \lim_{n \rightarrow \infty} (f(x_1) + f(x_2) + \cdots + f(x_n))\Delta x$$

$$= \frac{1}{b-a} \int_a^b f(x) dx$$



$$\text{Average value of } f \text{ over } [a, b] = \frac{1}{b-a} \int_a^b f(x) dx$$

**Ex 1.**

Find the average value of  $f(x) = \sin^3 x \cos x$  on the interval  $\left[0, \frac{\pi}{2}\right]$ .