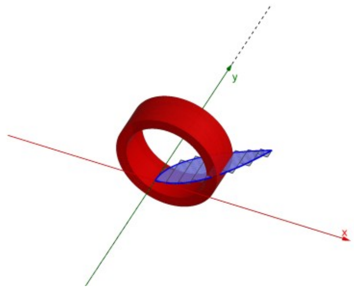
Volumes By Cylindrical Shells



Sometimes it's extremely difficult to use cross-sections.

Sometimes you have to use a different method that we'll call the "Shell Method" because our estimating solids will be ring-shaped cylindrical shells.

If we cut a shell, open it up, and lay it down flat, we get a thin box with volume:

$$2\pi(\text{radius}) \cdot (\text{height}) \cdot \Delta x$$

As the number of shells goes to infinity, the shells become infinitely thin. A finite sum of approximating volumes becomes an infinite sum using integration:

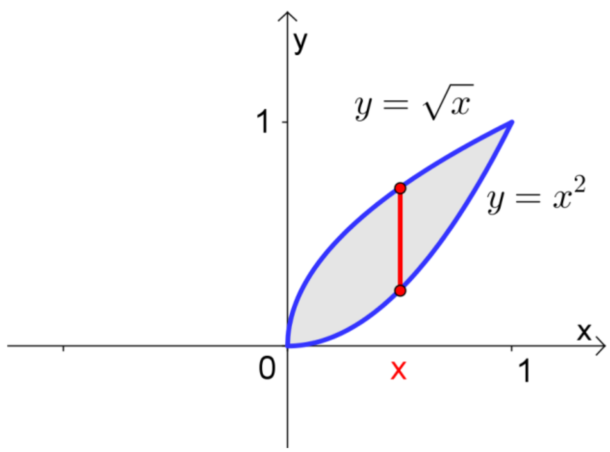
$$\boxed{V = \int_a^b 2\pi(\text{radius})(\text{height})\,dx}$$

(here, a is the left x-value of our region, and b is the right x-value)

Ex 1.

Find the volume of the solid generated by revolving the region bounded by the following curves about the y-axis.

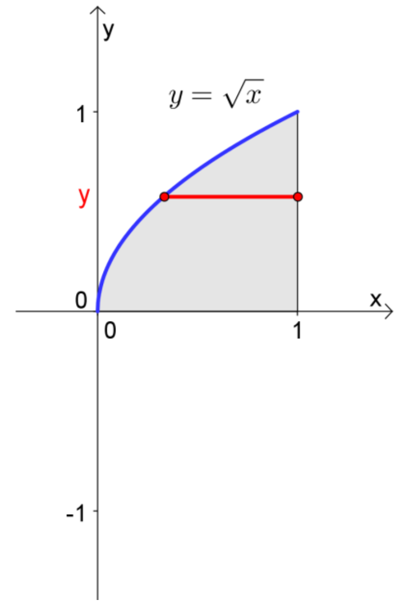
$y = \sqrt{x}, y = x^2$



Ex 2.

Find the volume of the solid generated by revolving the region bounded by the following lines and curves about the x-axis.

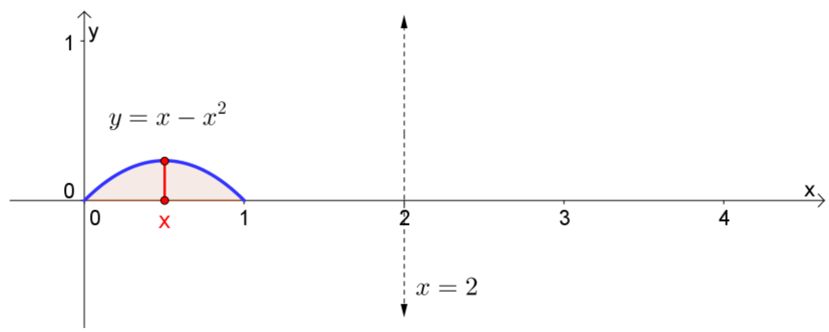
$y = \sqrt{x}, y = 0, x = 0, x = 1$



Ex 3.

Find the volume of the solid generated by revolving the region bounded by the following lines and curves about $x = 2$.

$y = x - x^2, y = 0$



Summary for volumes of revolution: washer (disk) ↔ perpendicular shell ↔ parallel

Practice

1. Find the volume of the solid generated by revolving the region bounded by the following lines and curves about $y = 1$. $x = y^2, x = 1$ (Use the shell method for practice.)

Q: What goes around the world but stays in a corner?