1. A solid lies between the planes perpendicular to the x-axis at $x = 0$ and $x = 2$. The cross-sections perpendicular to the x-axis between these planes are squares whose bases run from the curve $y = x^2$ to the curve $y = 2x$. Find the volume.

① Intersection points:

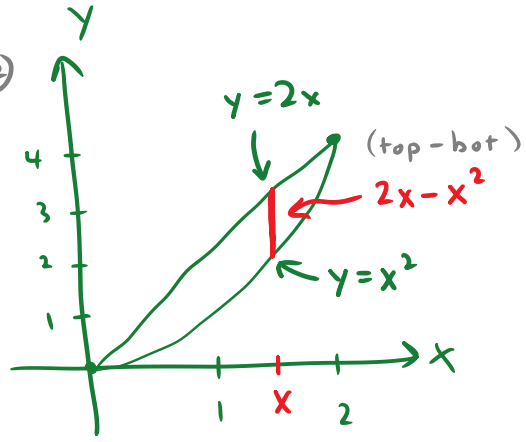
$x^2 = 2x$

$x^2 - 2x = 0$

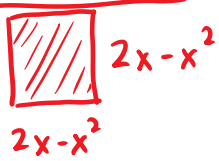
$x(x-2) = 0$

$x = 0 \quad x = 2$

$(0,0) \quad (2,4)$

② 

③ Cross-section:



$A(x) = (2x - x^2)^2$

④ $V = \int_0^2 A(x)\,dx$

$= \int_0^2 (2x - x^2)^2\,dx$

$= \int_0^2 (4x^2 - 4x^3 + x^4)\,dx$

$= \left[\frac{4}{3}x^3 - x^4 + \frac{1}{5}x^5\right]_0^2$

$= \left(\frac{4}{3}(2)^3 - (2)^4 + \frac{1}{5}(2)^5\right) - \left(\frac{4}{3}(0)^3 - (0)^4 + \frac{1}{5}(0)^5\right)$ (0)

$= \frac{32}{3} - 16 + \frac{32}{5}$

$= \frac{160}{15} - \frac{240}{15} + \frac{96}{15}$

$= \boxed{\frac{16}{15}}$

16
×15
80
160
240

2. Find the volume of the solid generated by revolving the region bounded by the curves below about the x-axis.

$y = 2\sqrt{x}, y = 2, x = 0$

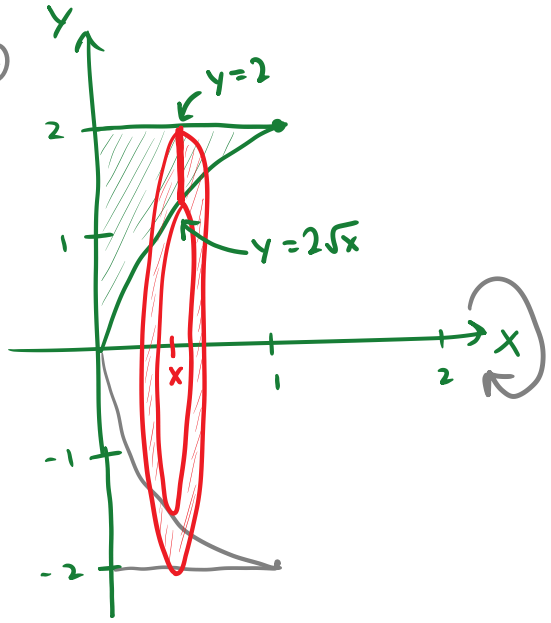
① Intersection pts:

$2\sqrt{x} = 2$

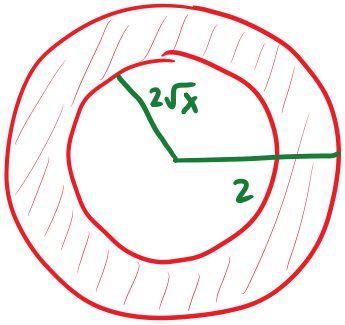
$\sqrt{x} = 1$

$x = 1$

$(1, 2)$

② 

③ Cross-section:



$A(x) = \pi(2)^2 - \pi(2\sqrt{x})^2$

$= 4\pi - 4\pi x$

$= 4\pi(1-x)$

④ $V = \int_0^1 A(x)\,dx$

$= \int_0^1 4\pi(1-x)\,dx$

$= 4\pi\left[x - \frac{1}{2}x^2\right]_0^1$

$= 4\pi\left(1 - \frac{1}{2}(1)^2 - (0 - \frac{1}{2}(0)^2)\right)$ (0)

$= \boxed{2\pi}$

Q: What can you catch, but not throw?