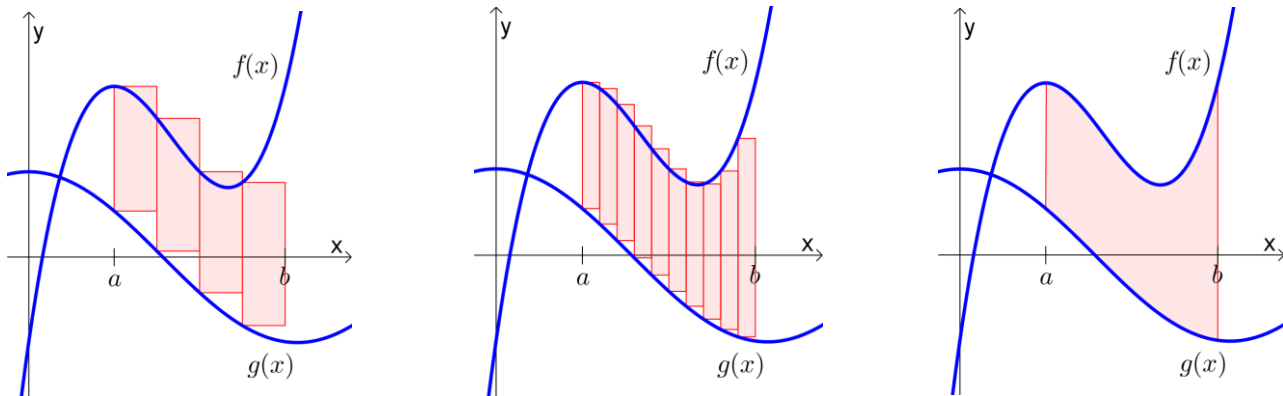


Area Between Curves

Q: How can we find the area between two curves?



$$\sum_{i=1}^n [f(x_i) - g(x_i)] \Delta x \xrightarrow{n \rightarrow \infty} \int_a^b [f(x) - g(x)] dx$$

If f and g are continuous with $f(x) \geq g(x)$ on $[a, b]$, then

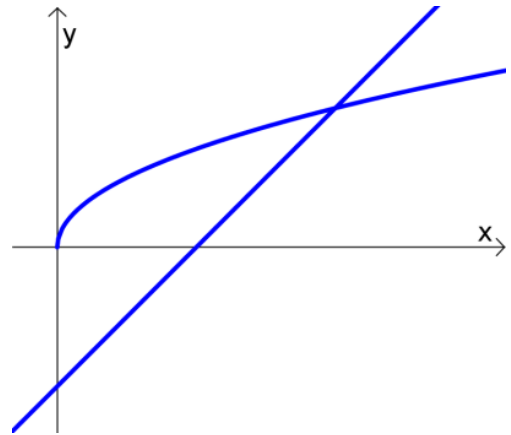
$$\text{Area between curves} = \int_a^b [f(x) - g(x)] dx$$

Ex 1.

Find the area of the region enclosed by the curves $y = 2 - x^2$ and $y = -x$.

Ex 2.

Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x -axis and the line $y = x - 2$.

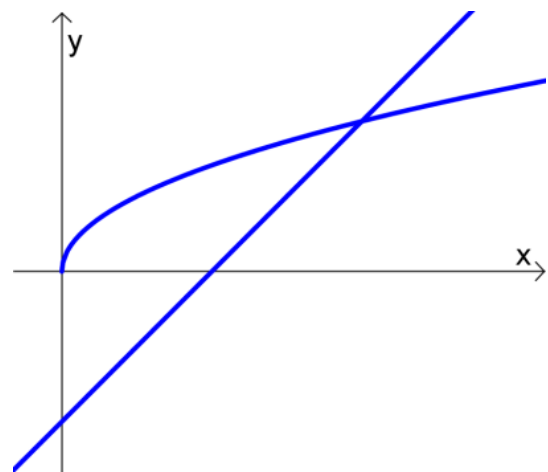


In the last example, it would have been more convenient to integrate with respect to y . The area formula becomes:

$$\text{Area between curves} = \int_c^d [f(y) - g(y)] dy$$

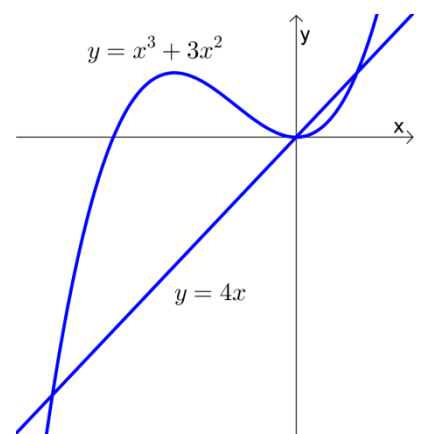
Ex 3.

Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x -axis and the line $y = x - 2$. But this time, integrate with respect to y .



Practice

1. Find the area of the region enclosed by the line $y = 4x$ and the curve $y = x^3 + 3x^2$.



2. Find the area of the region enclosed by the curves $x - y^2 = 0$ and $x + 2y^2 = 3$. (Hint: Integrate in y direction.)