

1. Write the first four terms of the sequence defined below. (This generates the **Fibonacci numbers**.)

$$a_1 = 1, \quad a_2 = 1, \quad a_{n+1} = a_n + a_{n-1}$$

$$a_1 = \boxed{1}$$

$$a_2 = \boxed{1}$$

$$a_3 = a_2 + a_1 = 1 + 1 = \boxed{2}$$

$$a_4 = a_3 + a_2 = 2 + 1 = \boxed{3}$$

2. Find a formula for the n th terms of the sequence $5, \frac{3}{2}, \frac{1}{3}, -\frac{1}{4}, -\frac{3}{5}, -\frac{5}{6}, -1, \dots$

← top: subtract 2

↖ bot: add 1

$$a_n = \frac{7-2n}{n}$$

3. Which of the following sequences converge, and which diverge? Find the limit of each convergent sequence.

a) $a_n = (-1)^n \left(1 - \frac{1}{n}\right)$

$\lim_{n \rightarrow \infty} (-1)^n \left(1 - \frac{1}{n}\right)$ does not exist (even terms approach 1, odd terms approach -1)

$\{a_n\}$ **diverges**

b) $a_n = 2 + (-1)^n$

$\lim_{n \rightarrow \infty} [2 + (-1)^n]$ does not exist (oscillates between 1 and 3)

$\{a_n\}$ **diverges**

c) $a_n = \frac{2n+1}{1-3\sqrt{n}}$

$$\lim_{n \rightarrow \infty} \frac{2n+1}{1-3\sqrt{n}} = \lim_{n \rightarrow \infty} \frac{2\sqrt{n} + \frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}} - 3} = -\infty$$

$\{a_n\}$ **diverges**

$$d) a_n = \left(\frac{2}{n}\right)^{1/n}$$

$$\ln a_n = \ln \left(\frac{2}{n}\right)^{1/n} = \frac{1}{n} \ln \left(\frac{2}{n}\right) = \frac{\ln 2 - \ln n}{n}$$

$$\lim_{n \rightarrow \infty} \ln a_n = \lim_{n \rightarrow \infty} \frac{\ln 2 - \ln n}{n} \stackrel{\text{L'Hosp}}{=} \lim_{n \rightarrow \infty} \frac{-1/n}{1} = 0$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} e^{\ln a_n} = e^0 = \boxed{1} \quad \boxed{\text{Converges}}$$

$$e) a_n = \frac{1 - \cos n}{n^2} = \frac{1}{n^2} - \frac{\cos n}{n^2}$$

$$\lim_{n \rightarrow \infty} \left(\frac{1}{n^2} - \frac{\cos n}{n^2} \right) = \lim_{n \rightarrow \infty} \frac{1}{n^2} - \lim_{n \rightarrow \infty} \frac{\cos n}{n^2}$$

$$\begin{aligned} -1 &\leq \cos n \leq 1 \\ -\frac{1}{n^2} &\leq \frac{\cos n}{n^2} \leq \frac{1}{n^2} \end{aligned}$$

Both $\rightarrow 0$,
so $\frac{\cos n}{n^2} \rightarrow 0$

$$= 0 - 0$$

$$= \boxed{0} \quad \boxed{\text{Converges}}$$

Q: What are the next two letters in this sequence: A E F H I K L M ?