11.10 – Practice

1. Find the Taylor series for $\frac{1}{x}$ at $x = 2$ using the definition of a Taylor series.

$f(x) = \frac{1}{x}$ $\qquad f(2) = \frac{1}{2}$

$f'(x) = -\frac{1}{x^2}$ $\qquad f'(2) = \frac{-1}{2^2}$

$f''(x) = \frac{2\cdot 1}{x^3}$ $\qquad f''(2) = \frac{2\cdot 1}{2^3}$

$f'''(x) = \frac{-3\cdot 2\cdot 1}{x^4}$ $\qquad f'''(2) = \frac{-3\cdot 2\cdot 1}{2^4}$

$f^{(4)}(x) = \frac{4\cdot 3\cdot 2\cdot 1}{x^5}$ $\qquad f^{(4)}(2) = \frac{4\cdot 3\cdot 2\cdot 1}{2^5}$

$f^{(n)}(x) = \frac{(-1)^n n!}{x^{n+1}}$ $\qquad f^{(n)}(2) = \frac{(-1)^n n!}{2^{n+1}}$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(2)}{n!}(x-2)^n = \sum_{n=0}^{\infty} \frac{(-1)^n \cancel{n!}}{\cancel{n!}\, 2^{n+1}}(x-2)^n$$

$$= \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x-2)^n}$$

(Note: If we didn't have to use the definition of a Taylor series, we could write:

$$\frac{1}{x} = \frac{1}{2+x-2} = \frac{1}{2\left(1-\left(\frac{-(x-2)}{2}\right)\right)}$$

$$= \frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{-(x-2)}{2}\right)^n$$ Geometric Series

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x-2)^n$$)

2. Find a Maclaurin series for $f(x) = x^2 \ln(1 + x^3)$ by using $\ln(1 + x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}$.

$$x^2 \ln(1+x^3)$$

$$= x^2 \cdot \sum_{n=0}^{\infty} \frac{(-1)^{n-1}(x^3)^n}{n}$$

$$= \boxed{\sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{n} x^{3n+2}}$$

Q: When can you add two to eleven and get one as the correct answer?