Curves Defined by Parametric Equations

Parametric equations can be used to describe curves in the xy-plane.

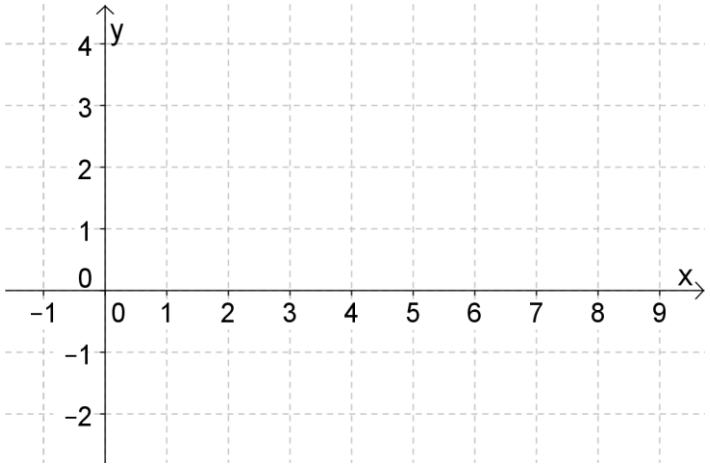
Here is an example of a set of parametric equations:

$x = t^2, \quad y = t + 1$

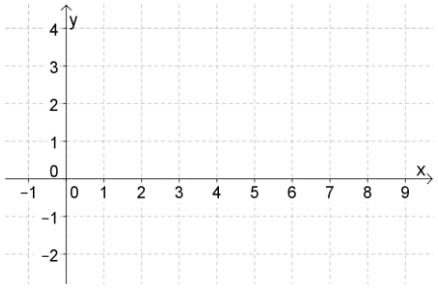
Each value of t gives you a point (x, y).

Let's plot a few points:

t	x	y
-3		
-2		
-1		
0		
1		
2		
3		



What would the graph look like if we restricted the **parameter interval** to $0 \leq t < 2$?



We could have turned the parametric equations $x = t^2,\ y = t + 1$ into a Cartesian equation:

Getting a Cartesian equation by eliminating the parameter helps us identify the path of the curve. However, eliminating the parameter is not even always possible (ex: $x = t - \ln t\,,\ y = t^2 + \sin t$).

Ex 1.

For the parametric equations $x = a\cos t$, $y = a\sin t$, $0 \le t \le 2\pi$, first eliminate the parameter to find a Cartesian equation of the curve. Then graph and describe the graph.

In the previous example, what would happen if we let our parameter interval be $0 \le t \le 4\pi$?

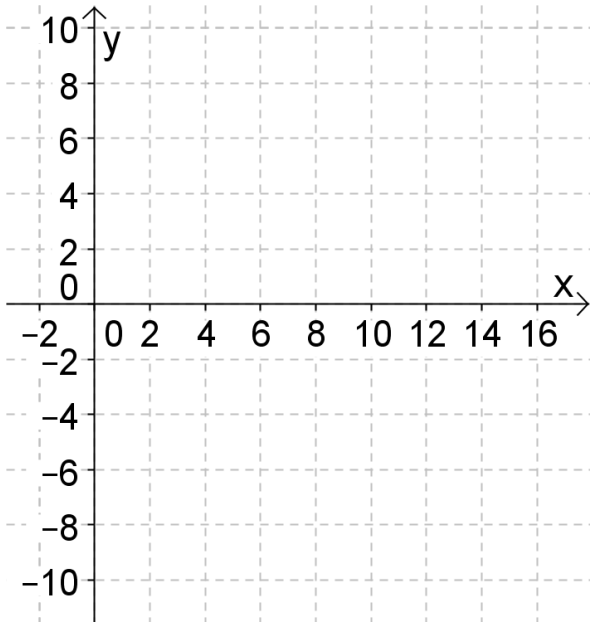
_____

Note: Any function $y = f(x)$ can be parametrized by $x = t,\ y = f(t)$.

For example, the parabola $y = x^2$, can be parametrized by _____.

Ex 2.

Graph $x = t + \frac{1}{t},\ y = t - \frac{1}{t},\ t > 0$.



Practice

1. Given the following parametric equations/intervals of a particle in the xy-plane, find the related Cartesian equation and graph it. Then, indicate the portion of the graph traced by the particle and the direction of motion.

   $x = -\sqrt{t},\ y = t,\ t \geq 0$

2. Given the following parametric equations/intervals of a particle in the xy-plane, find the related Cartesian equation and graph it. Then, indicate the portion of the graph traced by the particle and the direction of motion.

   $x = 4\sin t,\ y = 5\cos t,\ 0 \leq t \leq 2\pi$

3. Find a parametrization for the line segment with endpoints $(-1,3)$ and $(3,-2)$.

Q: A bus driver was heading down a street in Walnut. He went right past a stop sign without stopping, went the wrong way on a one-way street, and then went on the left side of the road past a cop car. The cop did nothing, because he didn't break any traffic laws. Why not?