_____ / 65 total points

Test #3

Math 181, Section 5, Prof. Beydler

Name: ________________________________

Wednesday, November 29, 2017

Directions: Show all work. No books or notes. A **scientific calculator** is allowed. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. Good luck!

1. (6 points) A tank contains 200 gal of fresh water. A solution containing 0.04 kg of salt per gallon of water runs into the tank at a rate of 5 gal/min. The mixture is kept thoroughly mixed and is pumped out of the tank at a rate of 5 gal/min. How many kilograms of salt will be in the tank after 10 minutes?

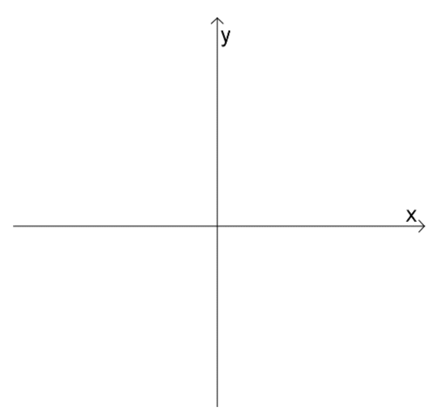
Answer: __________________________

2. (4 points) Solve the following initial value problem.
$y' = xy^2e^{2x},\ y(0) = 1$

$y =$ ____________________

3. (3 points) Given the following parametric equations/intervals of a particle in the xy-plane, find the related Cartesian equation and graph it. Then, indicate the portion of the graph traced by the particle and the direction of motion.
$x = e^t,\ y = e^{2t},\ 0 \le t \le 1$

Cartesian equation of curve: ____________________



4. (4 points) Find the area of the surface generated by revolving $x = \ln t\,,\ y = \ln t\,,\ 1 \leq t \leq e$ about the x-axis. Be sure to use calculus to get your answer.

Answer: ____________________________

5. (4 points) Set up but do not evaluate (an) integral(s) to find the area of the region…
…inside both $r = 2 + 2\cos\theta$ and $r = 3$.

Answer: ____

6. (6 points) Find the values of θ in $[0,2\pi)$ where the tangent line of $r = 1 + \sin\theta$ is horizontal or vertical. Be sure to use calculus and show your work!

Horizontal when $\theta =$ ____________________

Vertical when $\theta =$ ____________________

7. Determine whether each sequence converges or diverges. If it converges, find the limit.

a. (2 points) $a_n = \tan^{-1}\left(\ln\frac{1}{n}\right)$

Converges or diverges (circle one)

Limit (if convergent): ___________

b. (3 points) $a_n = n^{1/n}$ (be sure to show your reasoning)

Converges or diverges (circle one)

Limit (if convergent): ___________

8. (3 points) Assume that the following sequence converges and find its limit.
$a_1 = \sqrt{2}, \quad a_{n+1} = \sqrt{2a_n}$

Answer: ______________________

9. Determine whether each series is convergent or divergent. Be sure to state any test that you use and show your reasoning. If you use the Integral Test or Alternating Series Test, be sure to state the conditions of the test and (if necessary) show why the conditions are met.

a. (3 points) $\sum_{n=1}^{\infty} \frac{2}{\sqrt{n^5+n^3}}$

Convergent or divergent (circle one)

Test used: __________________________________

b. (4 points) $\sum_{n=1}^{\infty} \frac{2n+1}{n^2+2n+1}$

Convergent or divergent (circle one)

Test used: ___________________________________

c. (3 points) $\sum_{n=1}^{\infty} (-1)^n e^{1/n}$

Convergent or divergent (circle one)

Test used: ___________________________________

d. (4 points) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^2}{n^3+4}$

Convergent or divergent (circle one)

Test used: ___________________________________

10. Determine whether each series is convergent or divergent. If it converges, find its sum.

a. (4 points) $\sum_{n=1}^{\infty} \frac{2}{(n+1)(n+3)}$

Convergent or divergent (circle one)

Sum (if it converges): ____________________

b. (3 points) $\frac{1}{4} - \frac{3}{16} + \frac{9}{64} - \frac{27}{256} + \cdots$

Convergent or divergent (circle one)

Sum (if it converges): ____________________

11. Use $\sum_{n=1}^{\infty} \frac{1}{(n+2)^2}$ to answer the parts below.
   a. (3 points) How many terms are needed to make sure that the sum is accurate to within 0.0005?

Answer: ______________________

   b. (3 points) Use $\boxed{3}$ with $n = 4$ to give an estimate of the series' sum that's better than s_4. Write your answer to 6 decimal places.

Answer: ______________________________

12. (3 points) Find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ correct to 3 decimal places.

Answer: _______________________________

Here are the two integrals and the infamous [3] that I promised in case you need them.

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x \, dx = -\ln|\csc x + \cot x| + C$$

[3]: $\boldsymbol{s_n + \int_{n+1}^{\infty} f(x)\,dx \le \sum a_n \le s_n + \int_{n}^{\infty} f(x)\,dx}$