_____ / 15 total points

Quiz #3

Name: ________________________________

Math 181, Section 5, Prof. Beydler

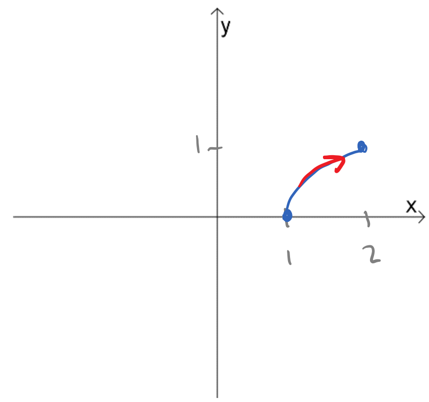
Wednesday, November 8, 2017

Directions: Show all work. No books or notes. A **scientific calculator** is allowed. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. Good luck!

1. (2 points) Eliminate the parameter to find a Cartesian equation of the curve. Then sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.
$x = t + 1, \;\; y = \sqrt{t}, \;\;\; 0 \le t \le 1$

$t = x - 1$

Cartesian equation of curve: $y = \sqrt{x-1}$



2. (3 points) Find the equation of the tangent line of the following curve at $t = 1$.
$x = t + \ln t\,, \;\; y = t^3$

$\frac{dx}{dt} = 1 + \frac{1}{t}$, $\frac{dy}{dt} = 3t^2$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{1 + \frac{1}{t}}$

$\left.\frac{dy}{dx}\right|_{t=1} = \frac{3(1)^2}{1 + \frac{1}{1}} = \frac{3}{2}$

At $t=1$:

$x = 1 + \ln 1 = 1$

$y = 1^3 = 1$

Equation of tangent line: $y - 1 = \frac{3}{2}(x - 1)$

3. (2 points) Show that $y = \cos 2x$ is a solution to the following initial-value problem.
$y'' + y' = -4y - 2\sin 2x, \quad y(0) = 1$

$LHS = y'' + y' = -4\cos 2x - 2\sin 2x$ =

$RHS = -4y - 2\sin 2x = -4\cos 2x - 2\sin 2x$

$y(0) = \cos 2(0) = 1$ ✓

4. (4 points) Solve the following initial-value problem. Be sure to explicitly solve for y as a function of x.
$\frac{dy}{dx} = \frac{x\sin x}{y^2}, y(0) = -1$

$y = \sqrt[3]{-3x\cos x + 3\sin x - 1}$

$\int y^2 dy = \int x \sin x\, dx$

$x \quad + \quad \sin x$
$1 \quad - \quad -\cos x$
$0 \quad \quad -\sin x$

$\frac{y^3}{3} = -x\cos x + \sin x + C$

Find C:

$\frac{(-1)^3}{3} = 0 + 0 + C$

$C = \frac{-1}{3}$

$\frac{y^3}{3} = -x\cos x + \sin x - \frac{1}{3}$

$y^3 = -3x\cos x + 3\sin x - 1$

5. (4 points) Find the orthogonal trajectories of the family of curves $y = kx^2$, where k is an arbitrary constant.
No need to solve explicitly for y.

Answer: $x^2 + 2y^2 = C$

$\frac{y}{x^2} = k$

$\frac{x^2 \frac{dy}{dx} - y \cdot 2x}{x^4} = 0$

$x^2 \frac{dy}{dx} - 2xy = 0$

$\frac{dy}{dx} = \frac{2y}{x}$

$\frac{dy}{dx} = -\frac{x}{2y}$

$\int 2y\, dy = \int -x\, dx$

$y^2 = -\frac{x^2}{2} + C$

$\frac{x^2}{2} + y^2 = C$