

Quiz #1

Name: _____

Math 181, Section 5, Prof. Beydler

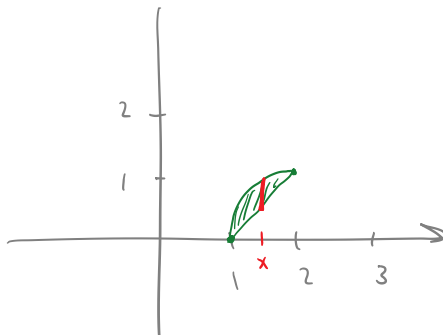
Wednesday, September 13, 2017

Directions: Show all work. No books or notes. A **scientific calculator** is allowed. Your desk and lap must be clear (no phones, notebooks, etc.). Write your answers in the indicated places, or box your answers. Good luck!

1. (5 points) Find the area of the region(s) enclosed by the curves $y = \sqrt{x-1}$ and $y = x-1$.

Int. pts:

$$\begin{aligned} \sqrt{x-1} &= x-1 \\ x-1 &= x^2-2x+1 \\ 0 &= x^2-3x+2 \\ 0 &= (x-1)(x-2) \\ x &= 1 \qquad x=2 \end{aligned}$$



Answer: 1/6

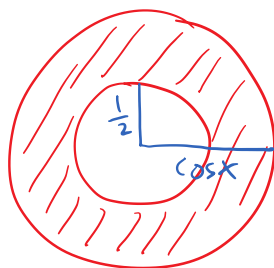
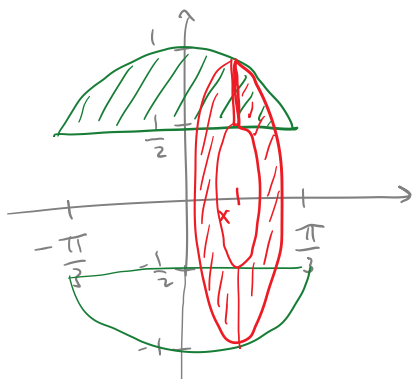
$$\begin{aligned} A &= \int_1^2 (\sqrt{x-1} - (x-1)) dx \\ &= \left[\frac{2}{3}(x-1)^{3/2} - \frac{x^2}{2} + x \right]_1^2 \\ &= \frac{2}{3}(2-1)^{3/2} - \frac{2^2}{2} + 2 - \left(\frac{2}{3}(1-1)^{3/2} - \frac{1^2}{2} + 1 \right) \\ &= \frac{2}{3} + \frac{1}{2} - 1 = \frac{1}{6} \end{aligned}$$

2. (2 points) A solid is generated by revolving the region bounded by $y = \cos x$, $y = \frac{1}{2}$, and $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$ about the x -axis. **Set up (but do not evaluate) an integral** to find the volume of the solid.

Int pts:

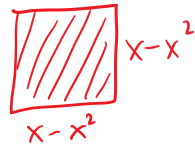
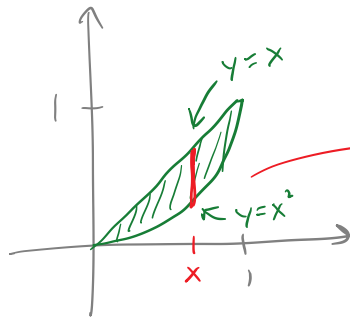
$$\begin{aligned} \cos x &= \frac{1}{2} \\ x &= -\frac{\pi}{3}, \frac{\pi}{3} \end{aligned}$$

Answer: $\int_{-\pi/3}^{\pi/3} [\pi(\cos x)^2 - \pi(\frac{1}{2})^2] dx$



3. (4 points) The base of a solid is the region between $y = x^2$ and $y = x$. Cross-sections perpendicular to the x -axis are squares with sides on the base. Find the volume of the solid.

Answer: $\frac{1}{30}$



$$\begin{aligned}
 V &= \int_0^1 (x - x^2)^2 dx \\
 &= \int_0^1 (x^2 - 2x^3 + x^4) dx \\
 &= \left[\frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5} \right]_0^1 \\
 &= \frac{1}{3} - \frac{1}{2} + \frac{1}{5} \\
 &= \frac{10}{30} - \frac{15}{30} + \frac{6}{30} \\
 &= \frac{1}{30}
 \end{aligned}$$

4. (4 points) A solid is generated by revolving the region bounded by $x = y^2 - y$ and $x = 2$ about the line $y = -2$. **Set up (but do not evaluate) an integral** to find the volume of the solid.

Int pts:

$$\begin{aligned}
 y^2 - y &= 2 \\
 y^2 - y - 2 &= 0 \\
 (y+1)(y-2) &= 0 \\
 y &= -1 \quad y = 2
 \end{aligned}$$

Answer: $\int_{-1}^2 2\pi (y+2) (2 - (y^2 - y)) dy$

\uparrow
 $y - (-2)$

