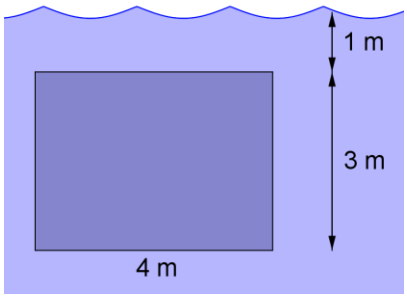
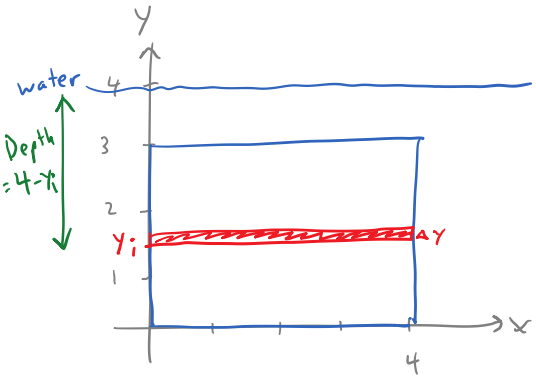
1. A vertical plate is submerged in water as shown. Use a Riemann sum to approximate the hydrostatic force against one side of the plate. Then find the exact hydrostatic force against one side of the plate.





$P_i = \rho g d = \left(1000 \frac{kg}{m^3}\right)\left(9.8 \frac{m}{s^2}\right)(4 - y_i \text{ m}) = 9800(4-y_i) \frac{N}{m^2}$

$A_i = 4\,\Delta y \leftarrow m^2$

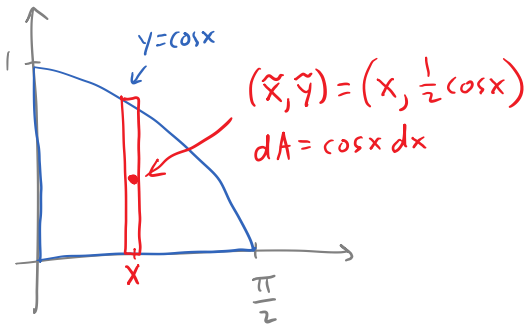
$F_i = P_i A_i = 39200(4-y_i)\Delta y \leftarrow N$

$$F = \int_0^3 39200(4-y)\,dy$$

$$= 39200\left[4y - \frac{y^2}{2}\right]_0^3$$

$$= 39200\left[4(3) - \frac{3^2}{2}\right] = \boxed{294000 \text{ N}}$$

2. Find the centroid of the region enclosed by $y = \cos x$, $y = 0$, $x = 0$, and $x = \frac{\pi}{2}$.



$$A = \int_0^{\frac{\pi}{2}} \cos x\,dx = [\sin x]_0^{\frac{\pi}{2}} = 1$$

$$\bar{x} = \frac{1}{1}\int_0^{\frac{\pi}{2}} \underbrace{x}_{\tilde{x}} \underbrace{\cos x\,dx}_{dA}$$

Int. by parts

x	+	$\cos x$
1	−	$\sin x$
0		$-\cos x$

$$= \left[x\sin x + \cos x\right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2} + 0 - (0 + 1)$$

$$= \frac{\pi}{2} - 1$$

$$\bar{y} = \frac{1}{1}\int_0^{\frac{\pi}{2}} \underbrace{\left(\frac{1}{2}\cos x\right)\cos x}_{\cos^2 x}\,dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{1+\cos 2x}{2}\,dx$$

$$= \left[\frac{1}{4}x + \frac{1}{8}\sin 2x\right]_0^{\frac{\pi}{2}} = \frac{\pi}{8}$$

$$\boxed{\left(\frac{\pi}{2} - 1, \frac{\pi}{8}\right)}$$

Q: What word starts with "e" and has only one letter in it?