Applications to Physics and Engineering

Hydrostatic Pressure and Force

The deeper you go underwater, the more the pressure.
Why? Because you have a taller and taller column of water above you, pushing down on you.

If we place a thin plate of area A horizontally underwater at a depth of d, then the volume of the column of water above it would be $V = Ad$.

The mass of the water would be $m = \rho V = \rho A d$ (where ρ is the mass-density of water).
So, the force of the water acting on the plate would be $F = mg = \rho g A d$.

Note that pressure (force per unit area) is $P = \frac{F}{A} = \frac{\rho g A d}{A} = \rho g d$.
So, you can also find force via pressure by the equation $F = PA$.

Notes:

- At any point of a given depth, the pressure applied in all directions is the same.
- $P = \rho g d$ shows that this pressure varies linearly with depth.
- For water, we'll use $\rho = 1000\ \mathrm{kg/m^3}$ and $\rho g = 62.5\ \mathrm{lb/ft^3}$.
- We can use the concept of a thin, flat plate to represent a surface (like the side of a swimming pool, the side of a trough, or the side of a dam).

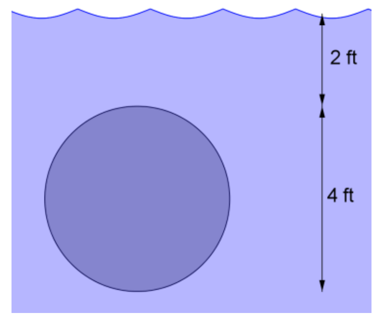
Ex 1.
A swimming pool full of water is 12 ft wide, 20 ft long, and 5 ft deep. What is the hydrostatic force acting on the bottom of the pool?

What if the plate is vertical instead? Then the depth is not constant anymore since it varies from the bottom to the top of the plate.

Ex 2.
What is the hydrostatic force acting on a 12-ft-by-5-ft side of the pool described in Ex 1?

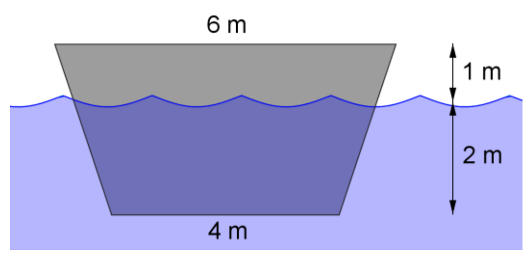
Ex 3.

A vertical plate is submerged in water as shown. Use a Riemann sum to approximate the hydrostatic force against one side of the plate. Then find the exact hydrostatic force against one side of the plate.

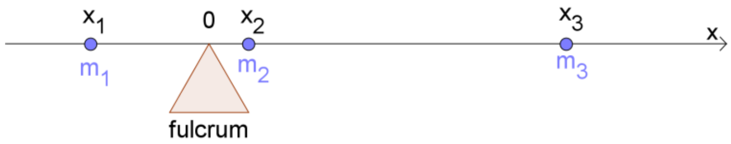


Ex 4.

Same directions as above.



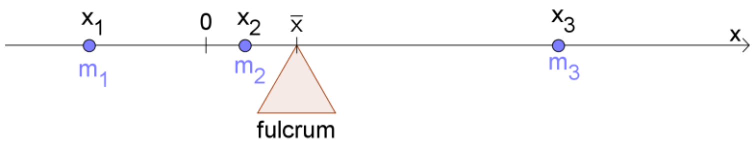
Center of Mass: One-dimension



______________ is the "turning effect" of a mass on a rigid body with a fulcrum.

The above system has torque: ____________________________ (Note: x_1, x_2, and x_3 are signed.)

We can change the torque of the system by moving the fulcrum. The torque is 0 when the fulcrum is placed at the _____________________________ (labeled $\overline{x}$). How can we calculate this x-value?



Note: $\sum m_i x_i$ is called the ______________ of the system about the origin (notice that it's just the torque of the system about the origin with acceleration removed). Note: $M = \sum m_i x_i$ and $m = \sum m_i$

Center of Mass: Two-dimensions

With masses on a plane (two-dimensions), our center of mass will have two coordinates: $(\overline{x}, \overline{y})$

$$\overline{x} = \frac{\sum m_i x_i}{\sum m_i} = \frac{M_x}{m} \qquad \overline{y} = \frac{\sum m_i y_i}{\sum m_i} = \frac{M_y}{m}$$

For a thin plate (lamina) that lies in a plane (like a circular or triangular sheet of metal), we can imagine cutting the plate into strips.

The center of mass of all the strips is again $(\overline{x}, \overline{y})$, but here

$$\overline{x} = \frac{\sum \tilde{x}_i \Delta m_i}{\sum \Delta m_i} \qquad \overline{y} = \frac{\sum \tilde{y}_i \Delta m_i}{\sum \Delta m_i}$$

With more and thinner strips, the finite sums become integrals, and we get the following formulas:

$$\overline{x} = \frac{\int \tilde{x}\, dm}{\int dm} \qquad \overline{y} = \frac{\int \tilde{y}\, dm}{\int dm}$$

Note: dm represents the mass of a strip, and can be found by multiplying the area density (ρ) by the area of the strip (dA).

When the area density ρ is uniform/constant, we have $\overline{x} = \frac{\int \tilde{x}\, dm}{\int dm} = \frac{\int \tilde{x}\, \rho dA}{\int \rho dA} = \frac{\rho \int \tilde{x}\, dA}{\rho \int dA} = \frac{\int \tilde{x}\, dA}{\int dA}$.

In this case, the center of mass is also called the ______________.

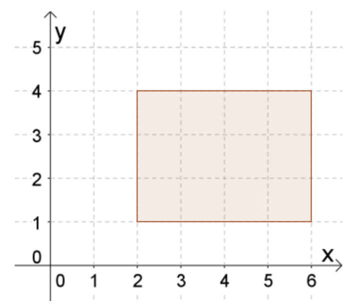
Note: $\int dA$ just represents the area of the plate/region/lamina, which we can just call A.

So, the centroid of a plate/region/lamina has the coordinates:

$$\boldsymbol{\overline{x} = \frac{1}{A}\int \tilde{x}\, dA} \qquad \boldsymbol{\overline{y} = \frac{1}{A}\int \tilde{y}\, dA}$$

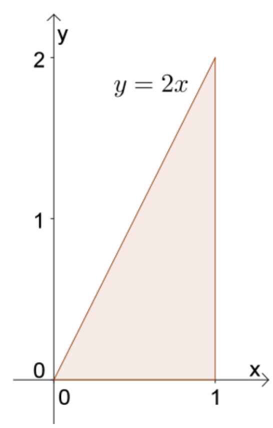
Ex 5.

Find the centroid of the rectangular plate shown to the right.



Ex 6.

Find the centroid of the triangular plate shown to the right.



Ex 7.

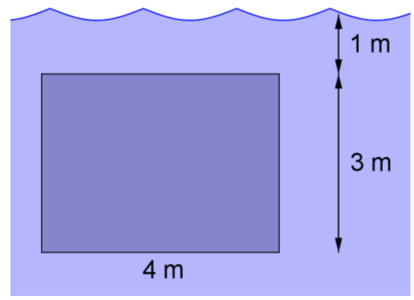
Find the centroid of the region enclosed by $y = x$ and $y = x^2$.

Ex 8.

Find the centroid of the region by $x = y^2 - y$ and $x = y$.

Practice

1. A vertical plate is submerged in water as shown. Use a Riemann sum to approximate the hydrostatic force against one side of the plate. Then find the exact hydrostatic force against one side of the plate.



2. Find the centroid of the region enclosed by $y = \cos x$, $y = 0$, $x = 0$, and $x = \frac{\pi}{2}$.

Q: What word starts with "e" and has only one letter in it?