

1. Evaluate:

$$\begin{aligned}
 \int_{-\infty}^2 \frac{2 dx}{x^2 + 4} &= \lim_{t \rightarrow -\infty} \int_t^2 \frac{2}{x^2 + 4} dx \\
 &= \lim_{t \rightarrow -\infty} \left[2 \cdot \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) \right]_t^2 \\
 &= \lim_{t \rightarrow -\infty} \left[\tan^{-1}\left(\frac{2}{2}\right) - \tan^{-1}\left(\frac{t}{2}\right) \right] \\
 &= \lim_{t \rightarrow -\infty} \left[\frac{\pi}{4} - \tan^{-1}\left(\frac{t}{2}\right) \right] \\
 &= \frac{\pi}{4} - \left(-\frac{\pi}{2}\right) \\
 &= \boxed{\frac{3\pi}{4}}
 \end{aligned}$$

2. Evaluate:

$$\begin{aligned}
 \int_0^4 \frac{dx}{\sqrt{4-x}} &= \lim_{t \rightarrow 4^-} \int_0^t (4-x)^{-1/2} dx \\
 &= \lim_{t \rightarrow 4^-} \left[-2(4-x)^{1/2} \right]_0^t \\
 &= \lim_{t \rightarrow 4^-} -2 \left(\sqrt{4-t} - \sqrt{4-0} \right) \\
 &= \lim_{t \rightarrow 4^-} -2 \left(\sqrt{4-t} - 2 \right) \\
 &= -2(0 - 2) \\
 &= \boxed{4}
 \end{aligned}$$

3. Does the following integral converge or diverge? (Use the Comparison Test.)

$$\int_4^{\infty} \frac{dx}{\sqrt{x}-1}$$

$$\frac{1}{\sqrt{x}-1} \geq \frac{1}{\sqrt{x}}$$

Since $\int_4^{\infty} \frac{1}{\sqrt{x}} dx$ diverges, ^($p = \frac{1}{2}$)

$\int_4^{\infty} \frac{dx}{\sqrt{x}-1}$ also diverges.

Q: What is it that you will break even when you name it?