1. Evaluate each integral.

a) $\int \cos^5 x\, dx$

$= \int (\cos^2 x)^2 \cdot \cos x\, dx$

$\cos^2 x = 1 - \sin^2 x$

$= \int (1-\sin^2 x)^2 \cdot \cos x\, dx$

$u = \sin x$

$du = \cos x\, dx$

$= \int (1-u^2)^2 \cdot du$

$= \int (1 - 2u^2 + u^4)\, du$

$= u - \frac{2}{3}u^3 + \frac{1}{5}u^5 + C$

$= \boxed{\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C}$

b) $\int \sin^2 x\, dx$

$= \int \frac{1-\cos 2x}{2}\, dx$

$= \frac{1}{2}\int (1 - \cos 2x)\, dx$

$= \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C$

$= \boxed{\frac{1}{2}x - \frac{1}{4}\sin 2x + C}$

c) $\int \tan^4 x\,dx$

$$= \int (\tan^2 x)^2\,dx$$

$$= \int (\sec^2 x - 1)^2\,dx$$

$$= \int (\sec^4 x - 2\sec^2 x + 1)\,dx$$

$$\int \sec^4 x\,dx = \int \sec^2 x \cdot \sec^2 x\,dx$$

$$= \int (\tan^2 x + 1)\cdot \sec^2 x\,dx \qquad u = \tan x,\ du = \sec^2 x\,dx$$

$$= \int (u^2+1)\,du$$

$$= \frac{u^3}{3} + u$$

$$= \frac{\tan^3 x}{3} + \tan x$$

$$= \frac{\tan^3 x}{3} + \tan x - 2\tan x + x + C$$

$$= \boxed{\frac{\tan^3 x}{3} - \tan x + x + C}$$

d) $\int \sqrt{\frac{1-\cos x}{2}}\,dx$ (Assume $0 \leq x \leq 2\pi$.)

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\sin^2\left(\frac{x}{2}\right) = \frac{1-\cos x}{2}$$

$$= \int \sqrt{\sin^2\left(\frac{x}{2}\right)}\,dx$$

Since $0 \leq x \leq 2\pi$, $\sin\left(\frac{x}{2}\right) \geq 0$

$$= \int \sin\left(\frac{x}{2}\right)dx$$

$$= \boxed{-2\cos\left(\frac{x}{2}\right) + C}$$

Q: The more you take, the more you leave behind. What are they?