Work

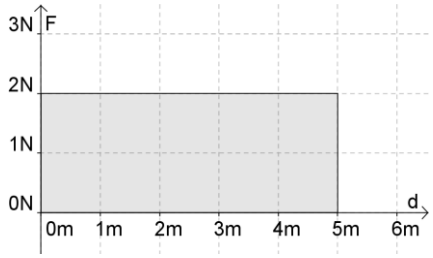
Work is done when a force F acts over a distance d, and is given by the equation:

$\boldsymbol{W = Fd}$

Work is measured using the following units:

SI units: Newton-meters ($N \cdot m$), also called joules (J)

British: Foot-pounds (ft-lb)



ex: If you apply a constant force of 2 Newtons over a distance of 5 meters, then you did

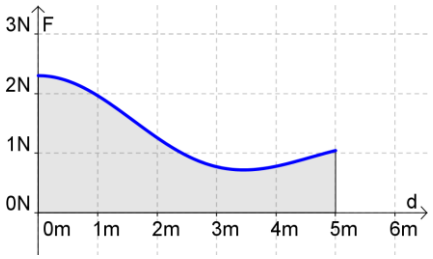
$W = Fd = (2\text{N})(5\text{m}) = 10\text{J}$

of work.

When the force varies, we integrate to find work:

$$\boldsymbol{W = \int_a^b F(x)\,dx}$$

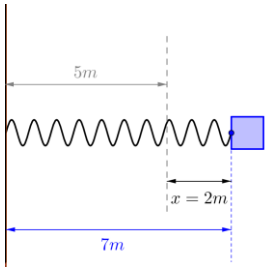
(Note: here, x represents an object's position)



Hooke's Law for Springs

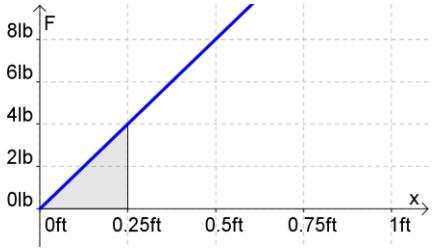
The force required to hold a stretched or compressed spring x units from its natural length is: $\boldsymbol{F = kx}$

(k is called the **force constant**, or **spring constant**, and has units N/m or lb/ft).



Ex 1.

Find the work required to compress a spring from its natural length of 1 ft to a length of 0.75 ft if the force constant is $k = 16$ lb/ft.



Ex 2.
A spring has a natural length of 1 m. A force of 24 N holds the spring stretched to a total length of 1.8 m. Find the force constant k.

How much work will it take to stretch the spring 2 m beyond its natural length?

How far will a 45-N force stretch the spring?

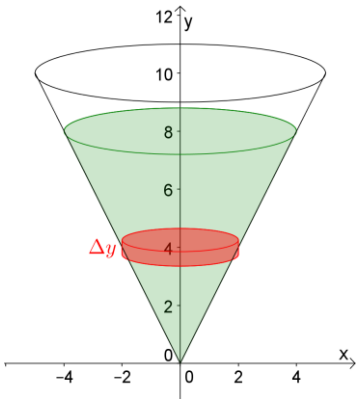
Pulling Up Ropes

Ex 3.
A 100-lb rope is 40 ft long and hangs vertically from the top of Building 61. How much work is required to lift the rope to the top of the building?

Pumping Liquids from Containers

Ex 4.

A conical tank with height 10 ft and base-radius 5 ft is filled to within 2 ft of the top with olive oil weighing 57 lb/ft^3. How much work does it take to pump the oil to the rim of the tank?



Note: Water has a density of $1000\ \text{kg/m}^3$ or $62.5\ \text{lb/ft}^3$.

Notes about units:

Using the metric system, the difference between mass and force is clear.

Typical units for mass are kg. Typical units for force are N, where 1 N = 1 kg · m/s^2.

To convert from mass to force, you'll use $F = ma$. For example, on the surface of the earth, the force due to gravity on a 5-kg object is $F = (5\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = 49\ \text{N}$. So a 5-kg object weighs 49 N.

But with British units, it's a little confusing. "lb" could mean mass (lbm) or force (lbf). On the surface of the earth, a 1-lbm object weighs approximately 1 lbf. However, on the moon a 1-lbm object weighs approximately 0.165 lbf. (Remember that mass doesn't depend on gravity. Weight however, does, because it's a force due to gravity.)

In these notes, "lb" means force. For example, $49\ \text{N} \approx 11.02\ \text{lb}$.

Practice

1. A force of 4 lb is required to hold a spring compressed 2 in. beyond its natural length. How much work is done in stretching the spring from its natural length to 5 in. beyond its natural length?

2. A rope with density 0.2 kg/m is hanging over the edge of building 61 so the end touches the ground. At the end of the rope, you attach a 5-kg tray and put in a 10-kg Calculus book. Assuming that building 61 is 30 m high, find the work done in pulling the rope, tray, and book up to the top of the building.

Q: What five-letter word becomes shorter when you add two letters to it?